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2082-3-28 Hints & Solution

Section – I

1.(b) Average velocity $= \frac{\text{Displacement}}{\text{Time}}$

$$= \frac{2R}{\left(\frac{2\pi R}{v}\right) \times \frac{1}{2}} = \frac{2v}{\pi}$$

2.(d) $E = \frac{1}{2} mv_0^2 = \frac{1}{2} m \frac{GM}{R} = \frac{GMm}{2R}$

Here, $E + E' = KE$ for escape

or, $E' = \frac{1}{2} mv_e^2 - E$

$= \frac{1}{2} m \times \frac{2GM}{R} - \frac{GMm}{2R}$

$= \frac{GMm}{2R} = E$

3.(b) $\frac{\text{Vol}}{\text{sec}} = \frac{\pi pr^4}{8\eta l}$

$\therefore \frac{(\text{Vol/sec})_{2nd}}{(\text{Vol/sec})_{1st}} = \left(\frac{r_2}{r_1}\right)^4 = \left(\frac{2r}{r}\right)^4 = 16$

4.(c) % rise = (Expansivity × $\Delta\theta$) × 100%

Here $\gamma > \beta > \alpha$ so, max. for volume

5.(c) Heat lost by water = Heat gained by ice

or, $50(100 - \theta) = 50 \times 80 + 50\theta$

or, $100 - \theta = 80 + \theta$

or, $\theta = \frac{20}{2} = 10°C$

6.(a) $m = -\frac{1}{2} = \frac{v}{u}$

or, $v = -\frac{u}{2}$

Now $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

or, $-\frac{1}{10} = \frac{1}{u} - \frac{2}{u} = -\frac{1}{u}$

$\therefore$ u = 10 cm

7.(c) $\frac{\lambda}{2} = 5\text{cm}$

$\lambda = 10 \text{ cm} = 0.1\text{m}$

$f = \frac{v}{\lambda} = \frac{2}{0.1} = 20 \text{ Hz}$

8.(c) $I_R = I_1 + 2\sqrt{I_1 I_2} \cos\phi + I_2$

$= 16a^2 + 2\sqrt{16a^2 \times a^2} \times \cos 60° + a^2$

$= 21a^2 = 21$ units

9.(a) $\theta_i - \theta_n = \theta_n - \theta_0$

or, $\theta_i = 2\theta_n - \theta_0 = 2 \times 300 - 10 = 590°C$

10.(b) $Q = \frac{d\phi}{R} = \frac{5.5 \times 10^{-4} - 5 \times 10^{-4}}{10} = 5 \times 10^{-6}C$

$= 5 \mu c$

11.(c) $I_{max} = \sqrt{I_1^2 + I_2^2}$

$I_{rms} = \frac{I_{max}}{\sqrt{2}} = \frac{1}{\sqrt{2}} (I_1^2 + I_2^2)^{1/2}$

12.(b) Intensity is independent of applied voltage.

13.(c) 'S' wave can travel from solid only.

14.(c) Potential at every point inside sphere is equal to potential on surface.

15.(c) First find the number of moles of CH_4 (molecular weight = 16 g/mol). Then multiply by Avogadro's number and remember that each CH_4 molecule contains 5 atoms (1 carbon + 4 hydrogen).

16.(b) Remember the flame test colors: Mg burns with bright white light, Ca gives brick red, Sr gives crimson red, and Ba gives apple green color.

17.(b) NH_3 acts as a proton acceptor (Brønsted base) but doesn't completely ionize in water, making it a weak base.

18.(c) Compare ionic compounds with covalent molecular compounds. Ionic bonds are much stronger than intermolecular forces like van der Waals forces.

19.(c) Consider how C_4H_{10} can be arranged - think about straight chain vs branched chain arrangements of carbon atoms.

20.(c) Look for a molecule with symmetrical geometry where the polar bonds cancel each other out due to the molecular shape.

21.(c) Color in transition metal ions arises from d-d electronic transitions. Ions with d^0 or d^{10} electronic configurations don't show d-d transitions.

22.(d) Atomic radius decreases from left to right across a period due to increasing nuclear charge pulling electrons closer.

23.(a) Elements in the same group have similar chemical properties because they have the same number of valence electrons.

24.(b) The period number is determined by the highest principal quantum number (n value) in the electron configuration.

25.(a) Among the hydrogen halides, acid strength increases down the group due to decreasing H-X bond strength, making it easier to release H^+.

26.(b) In $Cr_2O_7^{2-}$, chromium is in +6 oxidation state. When reduced to Cr^{3+}, each Cr atom gains 3 electrons. Remember there are 2 Cr atoms.

27.(b) In dilute acid solutions, water molecules are preferentially electrolyzed rather than the acid itself.

28.(a) The C = C double bond is electron-rich and readily attacks electrophiles, leading to a specific type of reaction.

29.(a) $x^2 = 16 \Rightarrow x = \pm 4$ and $2x = 6 \Rightarrow x = 3$

There is no value of x which satisfies both the above equations. Thus, $A = \phi$.

30.(d) $\left[\left(x^2 - 2 + \frac{1}{x^2}\right)^2\right]^{10} = \left(x^2 - 2 + \frac{1}{x^2}\right)^{20}$

$= \left[\left(x - \frac{1}{x}\right)^2\right]^{20} = \left(x - \frac{1}{x}\right)^{40}$

Total no. of terms after expansion = 40 + 1 = 41

31.(c) $|3AB| = 3^3|A|\,|B|$

$= 27 \times (-1) \times 2$

$= -54$

32.(c) $\sum_{n=1}^{\infty} \frac{1}{(2n-1)!} = \frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} + \ldots$

$= \frac{e - e^{-1}}{2}$

33.(a) If α and β are the roots, then

$A = \frac{\alpha + \beta}{2} \Rightarrow \alpha + \beta = 2A$

and $G = \sqrt{\alpha\beta} \Rightarrow \alpha\beta = G^2$

The quadratic eq^n is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\therefore \quad x^2 - 2Ax + G^2 = 0$

34.(c) $\tan 5\theta = \cot 2\theta = \tan\left(\frac{\pi}{2} - 2\theta\right)$

$\Rightarrow \quad 5\theta = n\pi + \frac{\pi}{2} - 2\theta$

$\Rightarrow \quad 7\theta = n\pi + \frac{\pi}{2} \Rightarrow \theta = (2n + 1)\frac{\pi}{14}$

35.(b) Let $\cot^{-1}x = \theta \Rightarrow \cot\theta = x$

Now, $\sin\cot^{-1}x = \sin\theta = \frac{1}{\text{cosec}\theta}$

$= \frac{1}{\sqrt{1 + \cot^2\theta}} = \frac{1}{\sqrt{1 + x^2}}$

36.(c) $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0 \Rightarrow a + b + c = abc + 2$

37.(c) $|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b}$

$= 1 + 1 + 0 = 2$

$\therefore \quad |\vec{a} + \vec{b}| = \sqrt{2}$

38.(d) $P(E) = \frac{5}{5 + 4} = \frac{5}{9}$

39.(b) $\lim_{x \to 0} \left(\frac{\frac{\sin x}{x} + \frac{\sin 3x}{x}}{\frac{\sin 3x}{x} - \frac{\sin x}{x}}\right)$

$= \lim_{x \to 0} \left(\frac{\frac{\sin x}{x} + \frac{\sin 3x}{3x} \times 3}{\frac{\sin 3x}{3x} \times 3 - \frac{\sin x}{x}}\right) = \frac{1 + 3}{3 - 1} = 2$

40.(c)

41.(b)

42.(d) Slope of normal $= -\frac{dx}{dy}$

$\Rightarrow \quad \tan\frac{3\pi}{4} = -\left(\frac{dx}{dy}\right)_{(3,\,4)}$

$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(3,\,4)} = 1$

i.e. $\quad f'(3) = 1$

43.(c) $\int_0^2 x^2[x]\,dx = \int_0^1 x^2(0)\,dx + \int_1^2 x^2(1)\,dx$

$= \left[\frac{x^3}{3}\right]_1^2 = \frac{7}{3}$

44.(c)

45.(b) Centroid divides the join of orthocentre and circumcentre in the ratio 2:1.

By section formula,

$(x, y) = \left(\frac{2.6 + 1.(-3)}{2 + 1}, \frac{2.2 + 1.5}{2 + 1}\right) = (3, 3)$

46.(a) Length of tangent = 5

i.e. $\quad \sqrt{5^2 + 4^2 + 2k.4} = 5$

Squaring $41 + 8k = 25$

$\therefore \quad k = -2$

47.(d) $y^2 + 2y + x = 0$

or, $\quad (y + 1)^2 = -(x - 1)$

$\therefore$ Vertex = (1, –1) which lies in 4th quadrant.

48.(d) $a^2 = 25 \Rightarrow a = 5$

$b^2 = 144 \Rightarrow b = 12$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 144}{5}$

$= \frac{288}{5}$

49.(a)	**50.(c)**	**51.(b)**	**52.(d)**	**53.(a)**	**54.(a)**
55.(c)	**56.(d)**	**57.(b)**	**58.(d)**	**59.(a)**	**60.(c)**

Section – II

61.(b) **1st cast**

$R = \sqrt{2^2 + 2 \times 2 \times 3\cos\theta + 3^2}$

$= \sqrt{13 + 12\cos\theta}$

2nd case

$2R = \sqrt{2^2 + 2 \times 2 \times 6\cos\theta + 6^2}$

$= \sqrt{40 + 24\cos\theta}$

Now, $2\sqrt{13 + 12\cos\theta} = \sqrt{40 + 24\cos\theta}$

or, $\quad 52 + 48\cos\theta = 40 + 24\cos\theta$

or, $\quad 12 = -24\cos\theta$

or, $\quad \cos\theta = -\frac{12}{24} = -\frac{1}{2} = \cos 120°$

$\theta = 120°$

62.(b) Impulse = Ft

= mat

= 0.15 × 20 × 0.1

= 0.3 NS

63.(a) $a = \frac{g\sin\theta}{1 + \frac{K^2}{r^2}}$

$= \frac{g\sin 30}{1 + \frac{1}{2}} = \frac{2}{3}g \times \frac{1}{2} = \frac{g}{3}$

64.(a) $\frac{1.004P}{P} = \frac{T + 1}{T}$

or, $\quad 1.004 = \frac{T + 1}{T}$

or, $\quad 1.004T = T + 1$

or, $\quad T = \frac{1}{0.004} = 250$ k

65.(b) In series

$\frac{Q}{t} = \frac{KAd\theta}{2l} = 2W$

In parallel,

$\left(\frac{Q}{t}\right)' = \frac{K2Ad\theta}{l} = 4\left(\frac{KAd\theta}{2l}\right)$

$= 4 \times 2 = 8W$

66.(d) $y = 8\sin 2\pi(0.1x - 2t)$

Now, $2\pi \times 0.1x = \frac{2\pi x}{\lambda}$

or, $\quad \lambda = \frac{1}{0.1} = 10$cm

$\therefore \quad \phi = \frac{2\pi x}{\lambda} = \frac{2\pi \times 2}{10} = \left(\frac{4 \times 180}{10}\right)^\circ = 72°$

67.(c) Length of diagonal

$l = \sqrt{3}\ a$

Distance of midpoint (r) $= \frac{l}{2} = \frac{\sqrt{3}a}{2}$

$V = 8 \times \frac{Q}{4\pi\varepsilon_0 r} = \frac{8Q \times 2}{4\pi\varepsilon_0 \times \sqrt{3}a}$

$= \frac{4Q}{\sqrt{3}\ \pi\varepsilon_0 a}$

68.(a) $I_1 = \frac{2}{6} = \frac{1}{3}$ A

$V_{AB} = I_1 R_{AB} = \frac{1}{3} \times 3 = 1V$

69.(a) $\eta = \frac{P_{out}}{P_{in}} \times 100\%$

or, $80\% = \frac{I_sE_s}{I_pE_p} \times 100\%$

$I_p = \frac{20 \times 120}{1000} \times \frac{100}{80} = 3A$

70.(a) $0 = \sqrt{I_1I_2}$

or, $3^2 = 9 \times I_2$

or, $I_2 = 1$ cm

71.(d) $\frac{\beta}{2} = \frac{D\lambda}{2d}$

$= \frac{1 \times 550 \times 10^{-9}}{2 \times 1.1 \times 10^{-3}}$

$= 2.5 \times 10^{-4}$ m

= 0.25 mm

72.(b) $\frac{m}{m_0} = \left(\frac{1}{2}\right)^{t/T_{1/2}}$

or, $m = m_0\left(\frac{1}{2}\right)^{t/T_{1/2}}$

$= 10.38\left(\frac{1}{2}\right)^{19/3.8}$

= 0.32g

73.(b) $\lambda_{particle} = \lambda$

or, $\frac{h}{m_pv_p} = \lambda$

or, $m_p = \frac{h}{v_p\lambda}$

$\frac{KE_{particle}}{E_{photon}} = \frac{\frac{1}{2}m_pv_p^2}{\frac{hc}{\lambda}}$

$= \frac{1}{2} \times \frac{h}{v_p\lambda}\frac{v_p^2}{hc} \times \lambda$

$= \frac{1}{2} = \frac{v_p}{c} = \frac{2.25 \times 10^8}{2 \times 3 \times 10^8}$

$= \frac{2.25}{6} = \frac{225}{600} = \frac{3}{8}$

74.(a) Count π bonds carefully. Remember that a triple bond consists of 1 σ bond + 2 π bonds, while a double bond has 1 σ bond + 1 π bond.

75.(b) Use the formula $2n^2$ where n is the shell number. For n=3, calculate $2(3)^2$.

76.(c) This reaction involves a hydrocarbon burning in oxygen to produce carbon dioxide and water, which is characteristic of a specific type of reaction.

77.(c) When H_2O_2 acts as a reducing agent, it gets oxidized (oxygen goes from -1 to 0). Look for a reaction where H_2O_2 reduces another species.

78.(b) First ionization energy generally increases across a period from left to right due to increasing nuclear charge, also ionization energy is affected by the electronic configuration.

79.(b) Notice that heat is released as a product in this reaction. This indicates the direction of energy flow.

80.(d) Volatility is inversely related to boiling point. Higher molecular weight alcohols have stronger intermolecular forces and higher boiling points.

81.(a) Count all single bonds carefully: C-H bonds, C-C single bonds. Remember that a triple bond contains 1 σ bond and 2 π bonds.

82.(a) $y = \sin^{-1}\left[\log_3\left(\frac{x}{3}\right)\right]$

$\Rightarrow\ -1 \le \log_3\left(\frac{x}{3}\right) \le 1$

$\Rightarrow\ \frac{1}{3} \le \frac{x}{3} \le 3$

$\Rightarrow\ 1 \le x \le 9$

$\therefore$ Domain = [1, 9]

83.(b) $\Sigma n = 55$

$\frac{n(n+1)}{2} = 55$

or, $n^2 + n - 110 = 0$

or, n = 10, –11 (not possible)

$\therefore$ n = 10

$\Sigma n^2 = \frac{n(n+1)(2n+1)}{6}$

$= \frac{10 \times 11 \times 21}{6}$

= 385

84.(a) Let $Z = \cos\frac{\pi}{8} + i\sin\frac{\pi}{8}$

Then $\frac{1}{Z} = \cos\frac{\pi}{8} - i\sin\frac{\pi}{8}$

Given expression $= \left(\frac{1+Z}{1+\frac{1}{Z}}\right)^8 = Z^8$

$= \left(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}\right)^8$

$= \cos\left(\frac{\pi}{8}\cdot 8\right) + i\sin\left(\frac{\pi}{8}\cdot 8\right)$

$= \cos\pi + i\sin\pi = -1$

85.(b) **Case I:** Ending at 0

Th H T U
↓ ↓ ↓ ↓
3 4 5 1 (Choices)

No. of ways = 3 × 4 × 5 × 1 = 60

Case II: Ending at 2 or 4

Th H T U
↓ ↓ ↓ ↓
4 4 3 2 (Choices)

No. of ways = 4 × 4 × 3 × 2 = 96

Required no. of ways = 60 + 96 = 156

86.(b) $\frac{\cos C + \cos A}{c + a} + \frac{\cos B}{b}$

$= \frac{b\cos C + b\cos A + c\cos B + a\cos B}{b(c + a)}$

$= \frac{a + c}{b(a + c)}$ [By projection law]

$= \frac{1}{b}$

87.(b) $\vec{a} + \vec{b} + \vec{c} = 0$

$\Rightarrow (\vec{a} + \vec{b} + \vec{c}).(\vec{a} + \vec{b} + \vec{c}) = 0$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$

$\Rightarrow \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = \frac{-1 - 4 - 9}{2} = -7$

88.(c) Given equations

$4X + 3Y + 7 = 0$ $\quad$ $3X + 4Y + 8 = 0$

$X = -\frac{3}{4}Y - \frac{7}{4}$ $\quad$ $Y = -\frac{3}{4}X - 2$

$\therefore b_{XY} = -\frac{3}{4}$ $\quad$ $b_{YX} = -\frac{3}{4}$

$r = -\sqrt{b_{XY}.b_{YX}} = -\frac{3}{4}$

89.(b) $\lim_{x \to 0} \frac{\sin(\pi \cos^2 x)}{x^2}$ $\left[\frac{0}{0} \text{ form}\right]$

$= \lim_{x \to 0} \frac{\cos(\pi \cos^2 x).(-\pi) \sin 2x}{2x}$

[By L' Hospital rule]

$= -\pi \lim_{x \to 0} \cos(\pi\cos^2 x). \lim_{x \to 0} \frac{\sin 2x}{2x}$

$= (-\pi)(-1)(1) = \pi$

90.(c) $y = \tan^{-1}\left(\frac{\cos x + \sin x}{\cos x - \sin x}\right)$

$= \tan^{-1}\left(\frac{1 + \tan x}{1 - \tan x}\right)$

$= \tan^{-1}\left(\frac{\tan\frac{\pi}{4} + \tan x}{1 - \tan\frac{\pi}{4}.\tan x}\right)$

$= \tan^{-1} \tan\left(\frac{\pi}{4} + x\right)$

$= \frac{\pi}{4} + x$

$\therefore \frac{dy}{dx} = 1$

91.(d) Here, $x = -2$. So, $x < 0$

Then, $|x| = -x$ as $x < 0$

$f(x) = |(-x)^2 - (-x)| = |x(x + 1)|$

$= x(x + 1)$ as $x < -1$

$= x^2 + x$

$f'(x) = 2x + 1$

$f'(-2) = -4 + 1 = -3$

Slope of normal $= -\frac{1}{\text{slope of tangent}} = \frac{1}{3}$

92.(b) Let $I = \int \frac{\cot x}{\ln \sin x} dx$

Put $\ln \sin x = t$

$\cot x \, dx = dt$

$I = \int \frac{1}{t} dt = \ln t + c = \ln(\ln \sin x) + c$

93.(b) $A = 2\int_0^1 y dx$

$= 2\int_0^1 (1 - x) dx$

$= 2\int_0^1 (1 - x) dx = 2\left[x - \frac{x^2}{2}\right]_0^1 = 2 \times \frac{1}{2} = 1$

94.(b) D.r's of the normal to the plane: 2, –3, 6

D.r's of x-axis: 1, 0, 0

$\sin\theta = \frac{2.1 + (-3).0 + 6.0}{\sqrt{2^2 + (-3)^2 + 6^2}\sqrt{1^2 + 0^2 + 0^2}}$

or, $\sin\theta = \frac{2}{7}$

or, $\theta = \sin^{-1}\left(\frac{2}{7}\right)$

or, $\sin^{-1}(\lambda) = \sin^{-1}\left(\frac{2}{7}\right)$

$\therefore \lambda = \frac{2}{7}$

95.(c) Let the slopes be m and 3m.

Then, $m + 3m = -\frac{2h}{b^2}$

$\Rightarrow 4m = -\frac{2h}{b^2}$

$\Rightarrow m = -\frac{h}{2b^2}$ .... (i)

and $m.3m = \frac{a^2}{b^2}$

or, $3\left(-\frac{h}{2b^2}\right)^2 = \frac{a^2}{b^2}$ [Using (i)]

$\therefore h = \frac{2}{\sqrt{3}} ab$

96.(b) $CD = \sqrt{(1 + 1)^2 + (0 - 2)^2 + (5 - 4)^2} = 3$

D.c's of CD:

$l = \frac{1 + 1}{3} = \frac{2}{3}$

$m = \frac{0 - 2}{3} = -\frac{2}{3}$

$n = \frac{5 - 4}{3} = \frac{1}{3}$

The projection of AB on CD

$= (x_2 - x_1) l + (y_2 - y_1) m + (z_2 - z_1)n$

$= (4 - 3)\frac{2}{3} + (6 - 4)\left(-\frac{2}{3}\right) + (11 - 5)\frac{1}{3}$

$= \frac{2 - 4 + 6}{3} = \frac{4}{3}$

97.(c) **98.(c)** **99.(c)** **100.(d)**

...Best of Luck...