Section – I

1.(a) Given equation is $x^2 + px + q = 0$

Let α and β are roots. Then $\alpha + \beta = -p$, $\alpha\beta = q$

Now, $\alpha^2 + \beta^2 = \alpha + \beta$

$(\alpha + \beta)^2 - 2\alpha\beta = \alpha + \beta$

$p^2 - 2q = -p$ i.e. $p^2 + p = 2q$

2.(a) $y = 1 + x + x^2 + \dots \text{ to } \infty = \dfrac{1}{1-x}$

or, $1 - x = \dfrac{1}{y}$ or, $x = 1 - \dfrac{1}{y}$

i.e. $x = \dfrac{y-1}{y}$

3.(c) Skew symmetric matrix has all principal diagonal elements zero and so their sum is 0.

4.(b) $4N = \{4, 8, 12, 16, 20, 24, 28, \dots\}$

$6N = \{6, 12, 18, 24, 30, \dots\}$

$4N \cap 6N = \{12, 24, \dots\} = 12N$

5.(a) Vectors $3\vec{i} + \vec{j} + \vec{k}$ and $\lambda\vec{i} + \lambda\vec{j} + \lambda\vec{k}$ are collinear

so $\dfrac{3}{\lambda} = \dfrac{1}{4} = \dfrac{1}{4}$ i.e. $\lambda = 12$

6.(b) Area $= \dfrac{1}{2} \times$ x-intercept $\times$ y-intercept

$= \dfrac{1}{2} \times 4 \times 5$

$= 10$ sq. units

7.(c) Circle is $x^2 + y^2 + 4x + 6y - 12 = 0$

Comparing with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get

$g = 2, f = 3, c = -12$

So length of intercepts on x-axis $= 2\sqrt{g^2 - c} = 2 \times 4 = 8$

8.(d) Focus = midpoint of latus rectum

$= \left(\dfrac{-1 + (-1)}{2}, \dfrac{5 + (-11)}{2}\right)$

$= (-1, -3)$

9.(d) $y = 0 \quad \dfrac{\lambda.3 + 1.(-1)}{\lambda + 1} = 0$

$3\lambda - 1 = 0 \quad \Rightarrow \lambda = \dfrac{1}{3}$

10.(c) $\text{cosec}^{-1}\text{cosec}\,\dfrac{5\pi}{4}$

$= \text{cosec}^{-1}\text{cosec}\left(\pi + \dfrac{\pi}{4}\right) = \text{cosec}^{-1}\left[-\text{cosec}\dfrac{\pi}{4}\right] = -\dfrac{\pi}{4}$

11.(d) $\text{Im}(z) = \dfrac{1}{2i}(z - \bar{z}) = -\dfrac{i^2}{2i}(z - \bar{z}) = \dfrac{1}{2}(\bar{z} - z)i$

12.(a) $r = \dfrac{\text{cov}(X, Y)}{\sqrt{\text{var}(X)}\sqrt{\text{var}(Y)}} = \dfrac{18}{\sqrt{16}\sqrt{81}} = 0.50$

13.(d) $P(A \cup B) = P(A) + P(B) - P(A)\,P(B)$

or, $0.61 = 0.48 + P(B) - 0.48\,P(B)$

or, $0.13 = 0.52\,P(B)$ $\therefore$ $P(B) = 0.25$

14.(a) The expansion is valid if $|3x| < 2$

i.e. $|x| < \dfrac{2}{3}$

i.e. $-\dfrac{2}{3} < x < \dfrac{2}{3}$

15.(a) $\lim\limits_{n \to \infty} \dfrac{1 + 2 + 3 + \dots + n}{n^2} = \lim\limits_{n \to \infty} \dfrac{n(n+1)}{2n^2} = \dfrac{1}{2}$

16.(b) $y = e^{3\log_e x} = x^3$ and $\dfrac{dy}{dx} = 3x^2$

17.(c) $\int e^x[f(x) + f'(x)]\,dx = e^x f(x) + c$

So, $\int e^x\left[\log_e x + \dfrac{1}{x}\right]dx = e^x \log_e x + c$

18.(b) By definition

19.(c) Checking $y(1) = 2$ which is 'C'

20.(a) Area enclosed $= \pi ab = \pi \times 5 \times 4 = 20\pi$ sq. units

21.(b) $K.E \propto (3t + 4)$

or, $F.S = K(3t + 4)$

or, $F.v = K \times 3$

$F = \dfrac{3K}{v} \propto \dfrac{1}{v}$

22.(a) In SHM, $K.E_{max} = P.E_{max} = K_0$

23.(a) $L = \vec{r} \times \vec{p}$

About origin, $\vec{r} = 0$ so $L = 0$

24.(d) $\dfrac{1}{F} = \dfrac{1}{f} + \dfrac{1}{(-f)} = 0$

$F = \infty$

25.(b) Sensitivity of potentiometer can be increased by increasing the potential gradient i.e. length of potentiometer wire.

26.(a) $r = \dfrac{\varepsilon}{I} = \dfrac{2}{4} = 0.5\ \Omega$

27.(d) $C = \varepsilon_r C_0$ $(C\uparrow)$

Again, $V = \dfrac{Q}{C}$ i.e. $V\downarrow$

28.(c) Equation of state is valid for any process

29.(d) $\phi = \dfrac{2\pi}{\lambda}x$

$x = \dfrac{\lambda}{2\pi}\phi = \dfrac{\lambda}{2\pi}\cdot\dfrac{\pi}{3} = \dfrac{\lambda}{6}$

30.(b) $r = \dfrac{mv}{qB} \propto \dfrac{v}{B}$

$\dfrac{r'}{r} = \dfrac{v'}{v}\cdot\dfrac{B}{B'}$

$= \dfrac{2v}{v}\cdot\dfrac{B}{\frac{B}{2}} = 4$

$r' = 4r$

31.(d) Sound can be identified by overtones.

32.(a) $E \propto Z^2$

$\frac{E'}{E_n} = \left(\frac{Z_2}{Z_1}\right)^2 = 4$

$E' = 4E_n$

33.(a) $\beta = \frac{\lambda D}{d} \propto \lambda$

$\because$ $\lambda_y > \lambda_b$, fringe width is greater for yellow than blue.

34.(d) $\frac{P'}{P} = \left(\frac{T'}{T}\right)^4 = \left(\frac{1.05T}{T}\right)^4 = 1.2155$

% increase $= \left(\frac{P'}{P} - 1\right) \times 100\%$

$= 21.55\%$

35.(b) 0.1 mole of $C_6H_{12}O_6$ = 18g

11.2 litres CO_2 at STP = 22g

N_A of CH_4 molecules = 16g

36.(c) Conjugate acid-base pair differ by a proton (H^+).

37.(c) 1 gm equivalent (i.e. 29.35 gm) of Ni is deposited by 1 Faraday of electricity.

0.1 F deposits 2.93 gm of Nickel.

38.(a) Strong acid + strong base ⟶ Normal salt

39.(a) $Z_{eff} = Z - \sigma$ (Z = atomic no. σ = shielding constant)

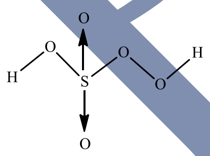
(Zeff = effective nuclear charge)

$Z_{eff} \propto = \frac{1}{\text{radius}}$

40.(d) Chloro-flurocarbons destroy ozone layer.

41.(c) At 300°C, $Na + NH_3 \rightarrow NaNH_2 + H_2$

42.(b)



43.(c) Hydrazine does not contain carbon.

44.(c) Both camphor and benzoic acid undergo sublimation so separated by chemical method.

45.(c) n-propyl alcohol = $CH_3 - CH_2 - \underset{}{\overset{OH}{\overset{|}{CH_2}}}$

iso-propyl alcohol = $CH_3 - \overset{OH}{\overset{|}{CH}} - CH_3$

46.(d) $CH_3C(CH_3)_2CH_3$ is known as neopentane.

47.(b) The compounds having $CH_3 - \overset{O}{\overset{||}{C}} -$ and $CH_3 - \overset{OH}{\overset{|}{CH}} -$ group gives iodoform test.

Ethyl alcohol (C_2H_5OH) has $CH_3 - \overset{OH}{\overset{|}{CH}} -$ group.

48.(d) Alkynes having triple bond on terminal carbon give Tollen's test but not silver mirror test.

49.d	**50.b**	**51.d**	**52.a**	**53.c**	**54.b**
55.d	**56.b**	**57.a**	**58.d**	**59.c**	**60.c**

Section – II

61.(c) a, b, c consecutive positive integers then a = b – 1, c = b + 1

So, $\log_e(1 + ac) = \log_e[1 + (b - 1)(b + 1)]$

$= \log_e(1 + b^2 - 1)$

$= \log_e b^2 = 2\log_e b$

62.(a) $\frac{1}{x+1} + \frac{1}{2(x+1)^2} + \frac{1}{3(x+1)^3} + \ldots$ to ∞

$= \frac{1}{x+1} + \frac{\left(\frac{1}{x+1}\right)^2}{2} + \frac{\left(\frac{1}{x+1}\right)^3}{3} + \ldots$ to ∞

$= -\log_e\left(1 - \frac{1}{x+1}\right) = -\log_e \frac{x+1-1}{x+1}$

$= -\log_e \frac{x}{x+1} = \log_e \frac{x+1}{x} = \log_e\left(1 + \frac{1}{x}\right)$

63.(b) $C(n, r + 1) + C(n, r - 1) + 2C(n, r)$

$= C(n, r + 1) + C(n, r) + C(n, r) + C(n, r - 1)$

$= C(n + 1, r + 1) + C(n + 1, r)$

$= C(n + 1 + 1, r + 1) = C(n + 2, r + 1)$

64.(c) Put $\alpha = \omega$ $\quad \beta = \omega^2$

Then $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 1$

$= \frac{\omega}{\omega^2} + \frac{\omega^2}{\omega} + 1 = \frac{\omega.\omega^3}{\omega^2} + \frac{\omega^2}{\omega} + 1$

$= \omega^2 + \omega + 1 = 0$

65.(d) Given $|\vec{a}| = |\vec{b}| = 1$

and $|\vec{a} + \vec{b}| = 1$

or, $(\vec{a} + \vec{b})^2 = 1$

or, $a^2 + b^2 + 2(\vec{a}.\vec{b}) = 1$

or, $1 + 1 + 2\vec{a}.\vec{b} = 1$

i.e. $2\vec{a}.\vec{b} = -1$

So, $|\vec{a} - \vec{b}| = \sqrt{(\vec{a} - \vec{b})^2} = \sqrt{a^2 + b^2 - 2\vec{a}.\vec{b}}$

$= \sqrt{1 + 1 + 1} = \sqrt{3}$

66.(c) Let the lines represented by $ax^2 + 2hxy + by^2 = 0$ be y = mx and y = 2mx. Then

$m + 2m = -\frac{2h}{b}$ i.e. $m = -\frac{2h}{3b}$

and $m.2m = \frac{a}{b}$ or, $2\left(-\frac{2h}{3b}\right)^2 = \frac{a}{b}$

or, $2.\frac{4h^2}{9b^2} = \frac{a}{b}$

$\therefore$ $8h^2 = 9ab$

67.(c) Given circle are $x^2 + y^2 - 4x - 6y - 12 = 0$, $x^2 + y^2 + kx + 4y - 12 = 0$

Here, $g_1 = 2, f_1 = 3, C_1 = -12$,

$g_2 = -\frac{K}{2}, f_2 = 2, C_2 = -12$

Two circles cut orthogonally if

$2g_1g_2 + 2f_1f_2 = C_1 + C_2$

or, $2.2\left(-\frac{K}{2}\right) = 2.(-3)(-2) = -12 - 12 \quad \therefore K = 6$

or, $-2K - 12 = -24$

or, $-2K = -12$

68.(b) Given parabola is $y^2 = 12x$. So, $a = 3$

Line is $x + y = K$ So, $m = -1, C = K$

So, line is a normal to parabola if $C = -2am - am^3$

i.e. $K = -2.3.(-1) - 3(-1)^3 = 6 + 3 = 9$

69.(c) Distance between directrices = 3 × distance between foci $2\frac{a}{e} = 3 \times 2ae$ or, $e^2 = \frac{1}{3}$ i.e. $e = \frac{1}{\sqrt{3}}$

70.(a) Given plane is $kx^2 + 6y^2 - 12z^2 + 6yz + 2zx + 7xy = 0$

Comparing it with $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$

We get, $a = K, b = 6, c = -12, f = 3, g = 1, h = \frac{7}{2}$

Since it represents a pair of planes so

or, $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

or, $K.6.(-12) + 2.3.1.\frac{7}{2} - K.3^2 - 6.1^2 - (-12).\left(\frac{7}{2}\right)^2 = 0$

or, $-72K + 21 - 9K - 6 + 12.\frac{49}{4} = 0$

or, $-81K + 15 + 147 = 0$

or, $81K = 162$ i.e. $K = 2$

71.(c) Here $\Delta = a^2 - (b - c)^2$

$= (a + b - c)(a - b + c)$

$= (2s - 2c)(2s - 2b)$

$= 4(s - b)(s - c)$

$s(s - a)(s - b)(s - c) = 16(s - b)^2(s - c)^2$

$\frac{1}{16} = \frac{(s - b)(s - c)}{s(s - a)}$

$\sqrt{\frac{1}{16}} = \sqrt{\frac{(s - b)(s - c)}{s(s - a)}}$

$\therefore$ $\tan\frac{A}{2} = \frac{1}{4}$

72.(d) $h(x) = f(x) + f(-x)$

So, $h'(x) = f'(x) - f'(-x)$

As h has an extrema, So $h'(x) = 0$ $f'(x) = f'(-x)$

$\therefore$ $f'(x)$ is an even function

73.(c) $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots \text{ to } \infty}}}$

or, $y = \sqrt{\sin x + y}$

or, $y^2 = \sin x + y$

On differentiation, $2y\frac{dy}{dx} = \cos x + \frac{dy}{dx}$

So, $\frac{dy}{dx} = \frac{\cos x}{2y - 1}$

74.(a) Put $y = \sqrt{x - \beta}$ $y^2 = x - \beta$

i.e. $x = y^2 + \beta$

$\therefore$ $dx = 2y\,dy$

Then $\int \frac{dx}{(x - \alpha)\sqrt{(x - \beta)}} = \int \frac{2y\,dy}{\sqrt{y^2 + \beta - \alpha}\,.\,y}$

$= 2\int \frac{dy}{\sqrt{y^2 + (\sqrt{\beta - \alpha})^2}}$

$= 2\ln\left(y + \sqrt{y^2 + \sqrt{(\beta - \alpha)^2}}\right) + c$

$= 2\ln\left(\sqrt{x - \beta} + \sqrt{x - \beta + \beta - \alpha}\right) + c$

$= 2\ln\left(\sqrt{x - \alpha} + \sqrt{x - \beta}\right) + c$

75.(b) Given curve is

$y = -x^2 + 2x + 3$

Putting $y = 0$, $x^2 - 2x - 3 = 0$

i.e. $x = -1, 3$

$\therefore$ Required area $= \int_{-1}^{3} (-x^2 + 2x + 3)\,dx$

$= \left[-\frac{x^3}{3} + x^2 + 3x\right]_{-1}^{3}$

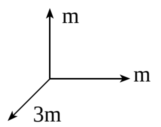
$= \left(-\frac{27}{3} + 9 + 9\right) - \left(\frac{1}{3} + 1 - 3\right)$

$= 9 + \frac{5}{3} = \frac{32}{3}$ sq. units

76.(a) $P = \sqrt{P_1^2 + P_2^2}$

$3m.v = \sqrt{(m \times 30)^2 + (m \times 30)^2}$

$v = \frac{30\sqrt{2}}{3} = 10\sqrt{2}$ m/s



77.(d) $P.A = 2\pi rT$

$\rho gh.\pi r^2 = 2\pi rT$

$h = \frac{2T}{\rho gr} = \frac{4T}{\rho gd} = \frac{4 \times 75 \times 10^{-3}}{1000 \times 10 \times 0.1 \times 10^{-3}} = 0.3\text{m}$

78.(b) $\Delta P.E = \left\{-\frac{GMm}{(R+R)} - \left(-\frac{GMm}{R}\right)\right\}$

$= \frac{GMm}{R}\left(1 - \frac{1}{2}\right)$

$= \frac{gR^2.m}{R} \cdot \frac{1}{2} = \frac{mgR}{2}$

79.(b) Gain in time in a day $= \frac{1}{2}\alpha\Delta\theta \times 1$ day

$\alpha = \frac{15 \times 2}{20 \times 86400} = 1.73 \times 10^{-5}/°C$

80.(b) $d = 2r = \frac{2h}{\sqrt{\mu^2 - 1}} = \frac{2 \times 1}{\sqrt{\left(\frac{4}{3}\right)^2 - 1}} = \frac{6}{\sqrt{7}}$

81.(a) $\varepsilon = B_H v l$

$= (0.18 \times 10^{-4}) \times 1 \times \frac{100 \times 1000}{3600}$

$= 0.5$ mV

82.(c) $n\left(\frac{1}{2}C_1V_1^2\right) = \frac{1}{2}C_2V_2^2$

$n = \frac{16 \times 10^{-6} \times (1000)^2}{8 \times 10^{-6} \times (250)^2}$

$= 2 \times 16 = 32$

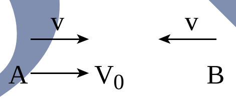
83.(a) $I = \frac{\text{total e.m.f.}}{\text{total resistance}} = \frac{10-6}{10} = 0.4A$

For 10V cell, $V = \varepsilon - Ir_1 = 10 - 0.4 \times 5 = 8V$

For 6V cell, $V = \varepsilon + Ir_2 = 6 + 0.4 \times 3 = 7.2$ V

84.(c) For A

$f' = \frac{V - V_0}{V} f$



For B

$f'' = \frac{V + V_0}{V} f$

$\therefore \quad f'' - f' = f_b$

$\frac{V + V_0}{V} f - \frac{V - V_0}{V} f = 10$

$\frac{2V_0}{V} f = 10$

$V_0 = \frac{10 \times 340}{2 \times 680} = 2.5$ m/s

85.(a) $Q = \int_0^{10} dQ = \int_0^{10} msd\theta = \int_0^{10} m.(0.6\theta^2)d\theta$

$= 10 \times 0.6 \left[\frac{\theta^3}{3}\right]_0^{10}$

$= 2 \times 10^3$ Cal

86.(b) Fraction left $\left(\frac{N}{N_0}\right) = \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$

$= \left(\frac{1}{2}\right)^{\frac{3 \times 60}{60}} = \frac{1}{8}$

% decayed $= \left(1 - \frac{1}{8}\right) \times 100\%$

$= 87.5\%$

87.(a) $\frac{\lambda'}{\lambda} = \frac{P}{P'} = \frac{\sqrt{2m \times \frac{3}{2}KT}}{\sqrt{2m \times \frac{3}{2}KT'}} = \sqrt{\frac{300}{1200}} = \frac{1}{2}$

$\lambda' = \frac{\lambda}{2}$

88.(d) Energy (E) $= hf_{max} = \frac{hc}{\lambda_{min}}$

$\lambda_{min} = \frac{hc}{E} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{40 \times 10^3 \times 1.6 \times 10^{-19}}$

$= 0.31 \times 10^{-10}$ m $= 0.31$ Å

89.(c)

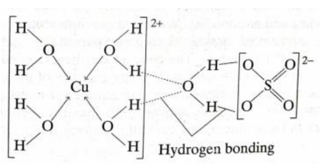
90.(c) Use $PV = nRT$ for getting n and number of molecules $= n \times 6.023 \times 10^{23}$.

91.(c) Minimum mol. wt. $= \frac{32 \times 100}{4}$. At least one S atom must be present.

92.(d) $Fe^{2+} \rightarrow Fe^{3+} + e$. In other species reduction (gain of electron occurs).

93.(d) N(+5) in $NO_3^- \rightarrow$ N(-3) in NH_3 i.e. 8*e*

94.(c)



95.(c) The compound dissolves in NaOH and gives characteristic colour with $FeCl_3$, hence, it is phenol on treatment with Br_2 it gives a tribromoderivative hence two ortho and one para position with respect to OH group must be free.

96.(d) Buta – 1, 2 – diene is $CH_2 = C = CH - CH_3$.

97.c **98.d** **99.b** **100.b**

…The End…