Section – I

1.(a) $y = A\cos\omega t$

or, $\frac{A}{2} = A\cos\left(\frac{2\pi}{T} \times t\right)$

or, $\cos 60° = \cos\left(\frac{2\pi}{T}.t\right)$

or, $\frac{\pi}{3} = \frac{2\pi}{T}.t$ or, $t = \frac{T}{6}$

2.(a) $u\cos\theta = \frac{1}{2}u$

or, $\theta = 60°$

3.(a) $U = \frac{1}{2} \times \text{stress} \times \text{strain}$

$= \frac{1}{2} \times (Y \times \text{strain}) \times \text{strain} = \frac{1}{2}Y\alpha^2$

4.(a) For linear source, $I \propto \frac{1}{r}$

i.e. $\frac{I'}{I} = \frac{r}{r'}$ or, $I' = \frac{r}{2r}I = \frac{I}{2}$

5.(b) $\Delta F = \frac{9}{5}\Delta C$

If $\Delta C = 1°C$, then $\Delta F = \frac{9}{5}°F$

$\alpha/°C = \alpha/\frac{9}{5}°F = \frac{5}{9}\alpha/°F$

6.(a) $\mu = A + \frac{B}{\lambda^2}$

Since 'λ' is longest for red light, its 'μ' is least.

7.(c) $\frac{R'}{R}\left(\frac{A}{A'}\right)^2 = \left(\frac{A}{\frac{A}{2}}\right)^2$

$R' = 4R$

$= 4 \times 10 = 40\ \Omega$

8.(b) Electron is retarded along the direction of electric field.

9.(b) $I_E = I_B + I_C$

or, $I_E = I_B + \beta I_B$

or, $I_E = (1 + \beta)I_B$

10.(a) $ms_w(\theta - 0) = mL_i$

or, $m \times 1 \times \theta = m \times 80$ or, $\theta = 80°C$

11.(b) $\cos\theta = \frac{R}{Z}$

At resonance Z = R

12.(c) $A = \sqrt{a^2 + a^2 + 2.a.a.\cos\pi} = 0$

13.(c)

14.(b) $\frac{\left(\frac{q}{m}\right)_\beta}{\left(\frac{q}{m}\right)_{H^+}} = \frac{\frac{e}{m_e}}{\frac{e}{m_p}} = \frac{m_p}{m_e} = \frac{1837\ m_e}{m_e} = 1837:1$

15.(a)

16.(b) FeS_2 (Iron pyrite)

O.N. of S = –1

O.N. of Fe = +2

17.(a)

18.(c)

19.(c)

20.(b) $NH_4NO_2 \rightarrow N_2 + 2H_2O$

21.(d) $IF_7 + H_2O \rightarrow HF + HIO_4$

(Per iodic acid)

22.(d) **23.(c)** **24.(a)** **25.(b)**

26.(a)

$H_2C = CH - \overset{\overset{\large O}{||}}{C} - H$

(Prop-2-en-al)
(Acraldehyde)

27.(c)

28.(c)

29.(c) $A \cap B = C$ implies $C \subseteq A$

$B \cap C = A$ implies $A \subseteq C$

So $C \subseteq A$ and $A \subseteq C$ implies $A = C$

30.(a) Fact $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

31.(d) Comparing $y^2 = 12x$ with $y^2 = 4ax$, we get a = 3

Focal distance = 4

$x + 3 = 4$

i.e. $x = 1$

32.(b) $\text{cosec}^2 x = \text{cosec}^2\theta$

$\Rightarrow \sin^2 x = \sin^2\theta$

$\Rightarrow x = n\pi \pm \theta$

33.(b) We know that A. adj A = |A| I

$|A|I = \begin{pmatrix} 9 & 0 \\ 0 & 9 \end{pmatrix} = 9\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 9I$

$\therefore |A| = 9$

34.(c) Given a, b, c are in GP so

$b^2 = ac$

$\Rightarrow \log_e b^2 = \log_e ac$

$\Rightarrow 2\log_e b = \log_e a + \log_e c$

$\Rightarrow \log_e a, \log_e b, \log_e c$ are in AP

$\Rightarrow \frac{1}{\log_e a}, \frac{1}{\log_e b}, \frac{1}{\log_e c}$ are in HP

$\Rightarrow \log_a e, \log_b e, \log_c e$ are in HP

35.(d) Equation is $x^2 + 5x + \lambda = 0$

Now, discriminant $= 5^2 - 4.1\lambda$

$= 25 - 4\lambda$

For imaginary roots, $25 - 4\lambda < 0$

i.e. $\lambda > 6.25$

So least integer is 7

36.(c) We know that $0 \le b_{xy} \times b_{yx} \le 1$

37.(b) Equation of rectangular hyperbola:

$x^2 - y^2 = a^2$

So eccentricity $= \sqrt{1 + \frac{a^2}{a^2}} = \sqrt{2}$

38.(c) We have $\sin^{-1}K + \cos^{-1}K = \frac{\pi}{2}$.

So $x + y = \frac{\pi}{2}$

39.(b) The vectors $3\vec{i} + \vec{j} - \vec{k}$ and $\lambda\vec{i} - 4\vec{j} + 4\vec{k}$ are collinear

So, $\frac{3}{\lambda} = \frac{1}{-4} = -\frac{1}{4}$ Then $\lambda = -12$

40.(c) Length of tangent = $\sqrt{5^2 + 2^2 - 4}$ = 5 units

41.(c) $\frac{de^{3\log_e x}}{dx} = \frac{de^{\log_e x^3}}{dx} = \frac{dx^3}{dx} = 3x^2$

42.(b) We know that $\int e^x[f(x) + f'(x)]\,dx = e^x f(x) + c$
Here f(x) = tanx f '(x) = $\sec^2 x$.
So, $\int e^x[\tan x + \sec^2 x]\,dx = e^x \tan x + c$

43.(c) Fact f(x) has local maximum value when f'(x) = 0, f''(x) < 0.

44.(c) Given eq^n is $4x^2 + 9y^2 = 36$
Here a = 3, b = 2
∴ Required area = $\pi ab = \pi.3.2$
= 6π sq. units

45.(b) Given differential equation is
$$\frac{d^3y}{dx^3} = \sqrt{1 + \left(\frac{d^2y}{dx^2}\right)^4}$$
i.e. $\left(\frac{d^3y}{dx^3}\right)^2 = 1 + \left(\frac{d^2y}{dx^2}\right)^4$
Here power of highest order derivative = 2 = degree of differential equation.

46.(b) The expansion is valid if |3x| < 2
i.e. $|x| < \frac{2}{3}$
i.e. $-\frac{2}{3} < x < \frac{2}{3}$

47.(c) Line parallel to y-axis makes an angle of 90° with x-axis.
So slope = tan90° = undefined.

48.(a) We have P(n, r) = C(n, r) × r!
336 = 36 × r! ⇒ r! = 6 ⇒ r = 3

49.d	**50.a**	**51.c**	**52.b**	**53.a**	**54.d**
55.b	**56.c**	**57.d**	**58.c**	**59.a**	**60.b**

Section – II

61.(a) $V_e = \sqrt{2gR}$
or, $V_e = \sqrt{2 \times \frac{4}{3}\pi R\rho G \times R}$ $\left(\because g = \frac{4}{3}\pi R\rho G\right)$
or, $V_e \propto R\sqrt{\rho}$
∴ $\frac{V_e'}{V_e} = \frac{R'\sqrt{\rho'}}{R\sqrt{\rho}}$
$\frac{V_e'}{V_e} = \frac{2R \times \sqrt{\frac{\rho}{4}}}{R\sqrt{\rho}}$
or, $V_e' = V_e$

62.(b) $\mu = \frac{F_f}{R} = \frac{F_f}{mg}$
or, $\mu = \frac{8}{2 \times 10} = 0.4$

63.(c) $f' = \frac{V + V_0}{V} \times f$
or, $f' = \frac{V + \frac{V}{5}}{V} \times f$ or, f ' = 1.2 f
∴ % increase
$= \left(\frac{f' - f}{f}\right) \times 100\%$
= 20%

64.(d) $N = N_0 e^{-\lambda t}$
or, $\frac{N}{N_0} = e^{t/T}$ $\left(\because \lambda = \frac{1}{T}\right)$
or, $\frac{N}{N_0} = \left(\frac{1}{e}\right)^{\frac{t}{T}}$
When t = 3T, then $\frac{N}{N_0} = \frac{1}{e^3}$
= 0.05
= 5%

65.(b) Work-done = change in K.E.
or, $\int_{x=20}^{x=30} F\,dx = K.E_f - K.E_i$
or, $\int_{20}^{30} (-0.2x)\,dx = K.E_f - \frac{1}{2}mu^2$
or, $-0.2\left(\frac{x^2}{2}\right)_{20}^{30} = K.E_f - \frac{1}{2} \times 10 \times 10^2$
∴ $50 = KE_f - 500$
or, $KE_f = 450$ J

66.(d) $\varepsilon = -L\frac{dI}{dt}$
or, $4 = L \times \frac{(1 - 0)}{1 \times 10^{-3}}$
or, $L = 4 \times 10^{-3}$ H

67.(d) K.E. = $-E_n$ = –(–3.4) = 3.4 eV
∴ $\lambda = \frac{h}{\sqrt{2mK.E}}$
$= \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 3.4 \times 1.6 \times 10^{-19}}}$
$= 6.65 \times 10^{-10}$ m

68.(a) $\frac{du}{dQ} = \frac{nc_v dT}{nc_p dT}$
or, $\frac{du}{dQ} = \frac{c_v}{c_p}$
or, $\frac{du}{dQ} = \frac{1}{\gamma} = \frac{5}{7}$ $(\because \gamma = c_p/c_v)$
$\gamma = \frac{7}{5}$ for diatomic gas

69.(c) $\beta = \frac{\lambda D}{d} = 10$ mm
$\beta' = \frac{\frac{\lambda}{\mu} \times D}{\frac{d}{2}}$
$= \frac{2}{\mu} \times \frac{\lambda D}{d} = \frac{2}{\frac{4}{3}} \times 10 = 15$ mm

70.(a) $Ig = \frac{V_G}{G} = \frac{5}{300} = \frac{1}{60}$ A

$\therefore\ S = \frac{IgG}{I - Ig} = \frac{\frac{1}{60} \times 300}{5 - \frac{1}{60}} \approx 1\Omega$ in parallel

71.(c) K.E. = P.E.

or, $\frac{1}{2} mv^2 = \frac{1}{4\pi\varepsilon_0} \frac{Qq}{r}$

i.e. $r \propto \frac{1}{V^2}$

$\frac{r'}{r} = \left(\frac{v}{v'}\right)^2 = \left(\frac{v}{2v}\right)^2 \quad \therefore\ r' = \frac{r}{4}$

72.(d) $\mu = \frac{c}{v}$

or, $\frac{\sin i}{\sin r} = \frac{c}{0.75C}$ ($\because$ v = 0.75C)

or, $\frac{i}{r} = \frac{1}{0.75}$ ($\because$ for small i, sini ≈ i)

or, r = 0.75i

Then, deviation (δ) = i – r

= i – 0.75i

$= \frac{1}{4}$ i

73.(d) $\frac{B_c}{B_a} = \frac{\mu_0 NI}{a} \times \frac{2(a^2 + a^2)^{3/2}}{\mu_0 NIa^2}$

$= \frac{2\sqrt{2}a^3}{a^3} = 2\sqrt{2} : 1$

74.(d) $4H_2C_2O_4 + 2KMnO_4 \rightarrow 8CO_2 + K_2O + Mn_2O_3 + 4H_2O$

Here, 63g $H_2C_2O_4$ = 61gm eq. wt.

100 ml 10 N $KMnO_4$=1000 ml 1N=1gm eq.wt

$\therefore$ CO_2 evolved = 1gm eq. wt = 22g(11.2 litres) (1/2 mole)

$\therefore$ xy = $\frac{1}{2} \times 11.2 = 5.6$

75.(b)

76.(a)

77.(c)

78.(a) The gas is hydrogen which reacts with Ca to form CaH_2 known as hydrolith

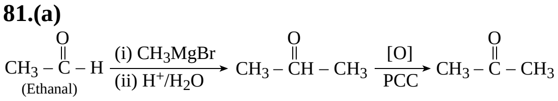
79.(b) $XeF_2 + 2H_2O \rightarrow 2Xe + O_2 + (4HF)$

$6XeF_4 + 12H_2O \rightarrow 2XeO_3 + 3O_2 + 4Xe + (24HF)$

$XeF_6 + 6H_2O \rightarrow XeO_3 + (6HF)$

80.(d)

81.(a)



82.(b) Six couples can be arranged in 6! ways.

Each couple canbe arranged in 2! = 2 ways

$\therefore$ Total number of arrangements

= 6! × 2 × 2 × 2 × 2 × 2 × 2

= $6! \times 2^6$

83.(c) Given $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

Now, $\vec{a} + \vec{b} + \vec{c} = \vec{0}$

or, $\vec{b} + \vec{c} = -\vec{a}$

or, $(\vec{b} + \vec{c})^2 = (-\vec{a})^2$

or, $b^2 + c^2 + 2\vec{b}.\vec{c} = a^2$

or, $1 + 1 + 2|\vec{b}|\,|\vec{c}| \cos\theta = 1$

or, $2.1 1.\cos\theta = -1$

or, $\cos\theta = -\frac{1}{2} \quad \therefore\ \theta = \frac{2\pi}{3}$

84.(b) Given $(1 + x)^n = c_0 + c_1x + c_2x^2 + ... + c_nx^n$

Differentiating,

$n(1 + x)^{n-1} = c_1 + 2c_2x + ... + nc_nx^{n-1}$

Putting x = 1, we have,

$n(1 + 1)^{n-1} = c_1 + 2c_2 + ... + nc_n$

$\therefore\ c_1 + 2c_2 + ... + nc_n = n2^{n-1}$

85.(a) f(x) = cos *l*nx

$f\left(\frac{x}{y}\right) + f(xy) = \cos ln\left(\frac{x}{y}\right) + \cos lnxy$

$= \cos(lnx - lny) + \cos(lnx + lny)$

$= 2\cos lnx \cos lny$

$= 2f(x) f(y)$

So, $f(x) f(y) - \frac{1}{2}\left[f\left(\frac{x}{y}\right) + f(xy)\right]$

$= f(x) f(y) - \frac{1}{2} . 2 f(x) f(y) = 0$

86.(b) There are 366 days in a leap year in which 52 weeks and 2 days. The possible combination for 2 days are (Sun, Mon), (Mon, Tue), (Tue, Wed), (Wed, Thu), (Thu, Fri), (Fri, Sat) and (Sat, Sun). Out of there of possibilities, 2 of then contains Sundays.

$\therefore$ Probability of 53 Sundays = $\frac{2}{7}$

87.(a) $x = \sqrt{-2 + 2\sqrt{-2 + 2\sqrt{-2 + ...}}}$

or, $x = \sqrt{-2 + 2x}$

or, $x^2 = -2 + 2x$

or, $x^2 - 2x + 2 = 0$

$\therefore\ x = \frac{2 \pm \sqrt{(-2)^2 - 4.1.2}}{2.1}$

$= \frac{2 \pm \sqrt{4 - 8}}{2} = \frac{2 \pm \sqrt{-4}}{2} = \frac{2 \pm 2i}{2}$

$= 1 \pm i$

88.(a) $f'(x) = x^2 + 2$

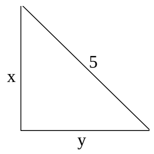
Integrating,

$f(x) = \frac{x^3}{3} + 2x + c$

$f(0) = 0 \Rightarrow c = 0$

So, $f(x) = \frac{x^3}{3} + 2x$

89.(b)



Here $x^2 + y^2 = 5^2$

When y = 3, x = 4 $\frac{dx}{dt} = -3$ m/s

So, $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$4x - 3 + 3\frac{dy}{dt} = 0$

i.e. $\frac{dy}{dt} = 4$ m/s

90.(d) Required area $= 2\int_0^1 (x - x^2)dx$

$= 2\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$

$= 2.\left(\frac{1}{2} - \frac{1}{3}\right)$

$= 2.\frac{1}{6} = \frac{1}{3}$ sq. units

91.(a) $y = \tan^{-1}\frac{4x}{1+5x^2} + \tan^{-1}\frac{2+3x}{3-2x}$

$= \tan^{-1}\frac{5x - x}{1 + 5x.x} + \tan^{-1}\frac{\frac{2}{3} + x}{1 - \frac{2}{3}.x}$

$= \tan^{-1}5x - \tan^{-1}x + \tan^{-1}\frac{2}{3} + \tan^{-1}x$

$= \tan^{-1}5x + \tan^{-1}\frac{2}{3}$

So, $\frac{dy}{dx} = \frac{5}{1 + (5x)^2} + 0 = \frac{5}{1 + 25x^2}$

92.(c) $I = \int \frac{\cot x}{\sqrt{\sin x}}dx$

Put y = sinx $\therefore$ dy = cosx dx

Then $I = \int \frac{\frac{\cos x}{\sin x}dx}{\sqrt{\sin x}} = \int \frac{dy}{y\sqrt{y}}$

$= \int y^{-3/2}dy$

$= \frac{y^{-1/2}}{-\frac{1}{2}} + c$

$= -\frac{2}{\sqrt{y}} + c$

$= -\frac{2}{\sqrt{\sin x}} + c$

93.(b) Hyperbola: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \;\therefore\; e = \sqrt{1 + \frac{b^2}{a^2}}$

i.e. $\frac{1}{e^2} = \frac{a^2}{a^2 + b^2}$

Conjugate hyperbola: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$

$\therefore e' = \sqrt{1 + \frac{a^2}{b^2}}$

i.e. $\frac{1}{e'^2} = \frac{b^2}{a^2 + b^2}$

So, $\frac{1}{e^2} + \frac{1}{e'^2} = \frac{a^2}{a^2 + b^2} + \frac{b^2}{a^2 + b^2} = 1$

94.(c) Eq^n: $ax^2 + 2hxy + by^2 = 0$

Here $m + 3m = -\frac{2h}{b}$

or, $4m = -\frac{2h}{b}$ i.e. $m = -\frac{h}{2b}$

and $3m = \frac{a}{b}$ or, $3m^2 = \frac{a}{b}$

or, $3.\frac{b^2}{4b^2} = \frac{a}{b}$ i.e. $\frac{h^2}{ab} = \frac{4}{3}$

95.(a) Here (α, β, γ)

$= \left(\frac{a + 0 + 0}{3}, \frac{0 + b + 0}{3}, \frac{0 + 0 + c}{3}\right)$

i.e. $a = 3\alpha$, $b = 3\beta$, $c = 3\gamma$

So, Eq^n of plane is

$\frac{x}{3\alpha} + \frac{y}{3\beta} + \frac{z}{3r} = 1$

i.e. $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$

96.(b) Given $\sec^{-1}x = \text{cosec}^{-1}y$

$\cos^{-1}\frac{1}{x} = \sin^{-1}\frac{1}{y}$

$\cos^{-1}\frac{1}{x} = \cos^{-1}\sqrt{1 - \left(\frac{1}{y}\right)^2}$

or, $\frac{1}{x} = \sqrt{1 - \frac{1}{y^2}}$

or, $\frac{1}{x^2} = 1 - \frac{1}{y^2}$

$\therefore \frac{1}{x^2} + \frac{1}{y^2} = 1$

97.d **98.c** **99.a** **100.c**

…The End…