Section – I

1.(b) $KE = \frac{1}{2}mv^2$

or, $\frac{\Delta KE}{KE} = \frac{1m}{m} + 2\frac{\Delta v}{v}$

$= 2\% + 2 \times 3\%$

$= 8\%$

2.(c) Body is in uniform speed in circular path so velocity changes due to direction.

3.(a) $F = \frac{dp}{dt} = 2bt$

$\therefore \quad F \propto t$

4.(a) wt = upthrust

$6g = \frac{V}{3}\sigma_w g$

or, $V = \frac{18}{\sigma_\omega}$ ….. (i)

Again, $(m + 6)g = V\sigma_w g$

or, $m = \frac{18}{\sigma_w}.\sigma_w - 6 = 12$ kg

5.(c)

6.(a) $S = \frac{dQ}{md\theta}$, for isothermal process $d\theta = 0$, $S = \infty$

7.(b)

8.(a) $\beta = \frac{D\lambda}{d}$, Here $\lambda_{yellow} > \lambda_{blue}$

So $\beta_{yellow} > \beta_{blue}$

9.(c)

10.(b) $f = \frac{2}{0.4} = 5$ beats/s

11.(a) Inside hollow spherical conductor, potential is equal to potential on surface.

12.(b) Q = CV, C is constant so V increases on increasing Q.

13.(d)

14.(a) $\frac{H_1}{H_2} = \frac{\frac{V^2}{R_1}}{\frac{V^2}{R_2}} = \frac{R_2}{R_1}$

15.(b) $H = B_e\cos\delta$

or, $Be = \frac{H}{\cos 30°} = \frac{2H}{\sqrt{3}}$

16.(d) $\lambda = \frac{h}{mv}$, m is least for β particle.

17.(c) $\alpha = 0.99 = \frac{\Delta I_c}{\Delta I_e}$

or, $\Delta I_c = 0.99 \times 5$

$= 4.95$ mA

18.(d)

19.(a)

20.(c)

21.(b) $SO_2 + 2H_2O \rightarrow H_2SO_4 + 2[H]$

22.(a) $CO + Cl_2 \xrightarrow{hv} COCl_2$ (Phosgene)

23.(c)

24.(c)

25.(b)

26.(c)

27.(d)

28.(a)

29.(d) Plotting the graphs, we have

As the graphs intersect at a point (0, 1).

So, $A \cap B \neq \phi$

30.(a) If AB – AC ⇒ B = C then A must be nonsingular as $A^{-1}AB = A^{-1}AC \Rightarrow B = C$

31.(a) As α and β are the imaginary cube roots of unity so, we may put $\alpha = \omega$, $\beta = \omega^2$. Then $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 1 = \frac{\omega}{\omega^2} + 1 = \frac{\omega^4}{w^2} + \frac{\omega^2}{\omega} + 1 = \omega^2 + \omega + 1 = 0$

32.(b) $\lim_{x \to \infty} x\sin\frac{1}{x} = \lim_{x \to \infty} \frac{\sin\frac{1}{x}}{\frac{1}{x}} = \lim_{y \to 0} \frac{\sin y}{y} = 1$

with $y = \frac{1}{x}$, $x \to \infty$ implies $y \to 0$

33.(b) x, y, z are in AP. So, –x, –y, –z are in A.P.

Then $-2y = (-x) + (-z)$

$\Rightarrow \quad e^{-2y} = e^{-x}.e^{-z} \Rightarrow (e^{-y})^2 = e^{-x}.e^{-z}$

$\Rightarrow \quad e^{-x}, e^{-y}, e^{-z}$ are in GP

34.(a) As the product $\sin^7x \cos^8x$ is odd.

So $\int_{-10}^{10} \sin^7x \cos^8x \, dx = 0$

35.(b) Replacing x by –x, we get $x^2 = 4ay$. So it is symmetric about y-axis.

36.(c) $t_{r+1} = c(12, r)\, (x^2)^{12-r}.\left(\frac{1}{x}\right)^r = c(12, r)\, x^{24-3r}$

For the term independent of x, we have $24 - 3r = 0 \Rightarrow r = 8$. So it is 9^{th} term.

37.(b) Equation of normal at (3, –4) is x(–u) = 3y

$\Rightarrow \quad 4x + 3y = 0$

38.(c) The equation of ellipse $3x^2 + 4y^2 = 12$

$\Rightarrow \quad \frac{x^2}{4} + \frac{y^2}{3} = 1$

So length of latus rectum $= 2\frac{b^2}{a} = 2.\frac{3}{2} = 3$

39.(b) As the vectors are orthogonal, so we have,

$(\vec{i} - 2\vec{j} + 4\vec{k}).(2\vec{i} + 7\vec{j} + m\vec{k}) = 0$

$\Rightarrow \quad 2 - 14 + 4m = 0$

$\Rightarrow \quad 4m = 12 \Rightarrow m = 3$

40.(c) Here $(\sqrt{2})^2 + (\sqrt{7})^2 = 2 + 7 = 9 = (3)^2$

So it is a right angled triangle

$\therefore$ Greatest angle = 90°

41.(b) Given $A = \tan^{-1}x$ i.e. $\tan A = x$.

So $\sin 2A = \frac{2\tan A}{1 - \tan^2 A} = \frac{2x}{1 - x^2}$

42.(a) For minimum value, $3x - 9 = 0$

$\Rightarrow \quad 3x = 9 \Rightarrow x = 3$

43.(b) $\sin x + \sin^2 x = 1 \quad \Rightarrow \sin x = 1 - \sin^2 x = \cos^2 x$

So, $\cos^2 x + \cos^4 x = \cos^2 x + (\cos^2 x)^2$

$= \sin x + \sin^2 x = 1$

44.(c) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = 2\cos^2\alpha - 1 + 2\cos^2\beta - 1 + 2\cos^2\gamma - 1$

$= 2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma) - 3 = 2 - 3 = -1$

45.(d) Here, coeff. of x^2 + coeff. of $y^2 = 3 + (-3) = 0$

So angle = 90°

46.(c) One root = 2 + 3i, other root = 2 – 3i.

So sum of roots = 4

$\Rightarrow \quad -\frac{p}{1} = 4 \quad \Rightarrow p = -4$

47.(d) $\log_{\sqrt{x}} x = \frac{1}{\log_x \sqrt{x}} = \frac{2}{\log_x x} = 2$

So its derivative is 0

48.(b) Given curve is $x^2 + y^2 = 16$.

On differentiation, we have $2x + 2y\frac{dy}{dx} = 0$

$\Rightarrow \quad \frac{dy}{dx} = -\frac{x}{y}$

For tangents perpendicular to y-axis, we have $\frac{dy}{dx} = 0 \Rightarrow x = 0$

Putting x = 0, we get $y^2 = 16 \Rightarrow y = \pm 4$

$\therefore$ Required points = (0, ±4)

49.(d)	50.(c)	51.(d)	52.(b)	53.(c)	54.(a)
55.(b)	56.(c)	57.(c)	58.(b)	59.(d)	60.(b)

Section – II

61.(b) For acceleration $v_{max} = at_1$

or, $t_1 = \frac{8}{a}$ ….(1)

For retardation

$0 = v_{max} - at_2$

or, $t_2 = \frac{8}{a}$ …. (2)

Now $t_1 + t_2 = \frac{8}{a} + \frac{8}{a}$

or, $4 = \frac{16}{a}$

or, $a = 4\ m/s^2$

So, $S_1 = \frac{1}{2}at_1^2$

$= \frac{1}{2} \times 4 \times 2^2 = 8m$

$S_2 = ut - \frac{1}{2}at^2$

$= 8 \times 2 - \frac{1}{2} \times 4 \times 2^2 = 8m$

$\therefore \quad S = S_1 + S_2 = 8 + 8 = 16\ m$

62.(d) Horizontal

$S_1 = vt = 1.5 \times 4 = 6\ m$

Vertical $a = \frac{F}{m} = \frac{5}{5} = 1\ m/s^2$

$S_2 = \frac{1}{2}at^2 = \frac{1}{2} \times 1 \times 4^2 = 8\ m$

$\therefore \quad S = \sqrt{S_1^2 + S_2^2} = \sqrt{6^2 + 8^2} = 10\ m$

63.(a) $F = \frac{YA_1e_1}{l_1} = \frac{YA_2e_e}{l_2}$

or, $\frac{\pi d^2}{4} \times \frac{10}{l} = \frac{\pi(2d)^2}{4} \times \frac{e_2}{2l}$

or, $e_2 = \frac{10}{2} = 5\ mm$

64.(d) $r = c_p - c_v = 525 - 315 = \frac{P_0}{\rho_0 T_0}$

or, $\rho_0 = \frac{P_0}{rT_0} = \frac{1.01 \times 10^5}{210 \times 273}$

$= 1.77\ kg/m^3$

65.(a)

66.(a) $\tan c = \frac{r}{h}$

or, $r = h\frac{\sin c}{\cos c}$

$= h \times \frac{1}{\mu} \times \frac{1}{\sqrt{1 - \sin^2 c}}$

$= \frac{h}{\sqrt{\mu^2 - 1}} = \frac{12}{\sqrt{\left(\frac{4}{3}\right)^2 - 1}}$

$= \frac{36}{\sqrt{7}}$ cm

67.(c) $f_A = 1.03f$
$f_B = 0.98f$
Now $f_A - f_B = 8$
or, $1.03f - 0.98f = 8$
or, $f = \frac{8}{0.05} = 160$ Hz
$\therefore$ $f_B = 0.98 \times 160$
$= 156.8$ Hz

68.(c) $F = K\frac{Q_1Q_2}{d^2}$ ….. (1)
2^{nd} case
$25\%\ F = K\frac{Q_1Q_2}{\left(\frac{d}{2} + \sqrt{K}\frac{d}{2}\right)^2}$
or, $\frac{F}{4} = \frac{4F}{(1 + \sqrt{K})^2}$
or, $1 + \sqrt{K} = 4$
or, $\sqrt{K} = 3$
or, $K = 9$

69.(b) $R_1 = \frac{24}{4} = 6\Omega$
$R_2 = \frac{3R}{4} = \frac{3 \times 24}{4} = 18\ \Omega$
$R_{eq} = \frac{R_1R_2}{R_1 + R_2} = \frac{6 \times 18}{6 + 18} = \frac{6 \times 18}{244} = 4.5\ \Omega$
$R = R_{eq} + r = 4.5 + 1.5 = 6\ \Omega$
$I = \frac{E}{R_T} = \frac{18}{6} = 3A$
$I' \times 18 = (I - I')^6$
or, $4I' = 3$
or, $I' = 0.75$ A

70.(c) Flux due to wire
$B_w = \frac{\mu_0 I}{2\pi a} = \frac{4\pi \times 10 \times 30}{2\pi \times 0.02}$
$= 3 \times 10^{-4}$ T
External field $B = 4 \times 10^{-4}$ T
Two fields are $\perp$ to each other so
$B_R = \sqrt{B_w^2 + B^2} = \sqrt{(3 \times 10^{-4})^2 + (4 \times 10^{-4})^2}$
$= 5 \times 10^{-4}$ T

71.(b) $E_0 = BA\omega N$
$= 0.05 \times 80 \times 10^{-4} \times 2\pi\left(\frac{2000}{60}\right) \times 50$
$= \frac{4\pi}{3}$ V

72.(c) 1^{st} case
$KE = h \times 2f_0 - hf_0 = hf_0$
or, $\frac{1}{2}mv^2 = hf_0$ ….. (1)
2^{nd} case
$KE' = h \times 5f_0 - hf_0 = 4hf_0$
or, $\frac{1}{2}mv'^2 = 4hf_0$ …. (2)
Dividing (2) by (1)
$\left(\frac{v'}{v}\right)^2 = \frac{4}{1}$
or, $v' = 2 \times 4 \times 10^6 = 8 \times 10^6$ m/s

73.(d) For x_1
$N_{x_1} = N_0e^{-10\lambda t}$ … (1)
For x_2
$N_{x_2} = N_0e^{-\lambda t}$ …. (2)
$\frac{N_{x_1}}{N_{x_2}} = \frac{1}{e}$
or, $\frac{e^{-10\lambda t}}{e^{-\lambda t}} = \frac{1}{e}$
or, $\left(\frac{1}{e}\right)^{9\lambda t} = \left(\frac{1}{e}\right)^1$
or, $9\lambda t = 1$
or, $t = \frac{1}{9\lambda}$

74.(b) $\theta = \frac{\beta}{2D} = \frac{D\lambda}{2dD} = \frac{\lambda}{2d}$
$= \left(\frac{5460 \times 10^{-10}}{2 \times 0.1 \times 10^{-3}} \times \frac{180}{\pi}\right)^\circ$
$= 0.16^\circ$

75.(c) The value of m ranges ranges from $-l$ to $+l$ including zero.

76.(c) $KClO_3 \xrightarrow{\Delta} K_2O + ClO_2$

77.(a)

78.(c) Methane on electrolysis gives ethyne (C_2H_2)

79.(b) $Fe^{2+} + 2e^- \longrightarrow Fe$, $E_{Fe} = \frac{56}{2} = 28$
$W_{Fe} = E_{Fe} \times$ No. of Faraday
$28 \times 3 = 84$

80.(a) $N_{mix} = \frac{75 \times 0.2 + 10 \times 0.5 + 30 \times 0.1}{75 + 10 + 30} = 0.2N$

81.(a) $2NaHCO_3 \xrightarrow{\Delta} Na_2CO_3 + H_2O + CO_2$
2 moles $NaHCO_3$ gives 1 mole Na_2CO_3
0. 2 moles $NaHCO_3$ gives $\frac{1}{2} \times 0.2 = 0.1$ mole

82.(a) $\int\left(\frac{2 + \sin2x}{1 + \cos2x}\right)e^x dx = \int\frac{2 + 2\sin x\cos x}{2\cos^2 x}e^x dx$
$= \int\left(\frac{1}{\cos^2 x} + \frac{\sin x\cos x}{\cos^2 x}\right)e^x dx$
$= \int(\sec^2 x + \tan x)\,e^x dx = e^x\tan x + c$
As it is of the form $\int e^x(f(x) + f'(x))\,dx = e^x f(x) + c$

83.(d) Given $y = \cos x$
$\frac{dy}{dx} = -\sin x = \cos\left(\frac{\pi}{2} + x\right)$

$\frac{d^2y}{dx^2} = -\cos x = \cos(\pi + x) = \cos\left(2.\frac{\pi}{2} + x\right)$

$\frac{d^3y}{dx^3} = \sin x = \cos\left(3\frac{\pi}{2} + x\right)$

and so on. So $\frac{d^ny}{dx^n} = \cos\left(\frac{n\pi}{2} + x\right)$

84.(b) Given $4\sin^{-1}x + \cos^{-1}x = \pi$

or, $3\sin^{-1}x + \sin^{-1}x + \cos^{-1}x = \pi$

or, $3\sin^{-1}x + \frac{\pi}{2} = \pi \Rightarrow 3\sin^{-1}x = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}x = \frac{\pi}{6}$

$\Rightarrow x = \sin\frac{\pi}{6} = \frac{1}{2}$

85.(b) Here, $\frac{a^2\sin(B-C)}{\sin B + \sin C} = \frac{4R^2\sin^2A\sin(B-C)}{\sin B + \sin C}$

$= \frac{4R^2\sin A\sin(B+C)\sin(B-C)}{\sin B + \sin C}$

$= \frac{4R^2\sin A(\sin^2B - \sin^2C)}{(\sin B + \sin C)}$

$= 4R^2\sin A(\sin B - \sin C)$

So, $4R^2[\sin A(\sin B - \sin C) + \sin B(\sin C - \sin A) + \sin C(\sin A - \sin B)] = 0$

86.(c) Here, $\frac{-1+\sqrt{3}i}{2} = \omega$, $\frac{-1-\sqrt{3}i}{2} = \omega^2$

$n = 3k + 1, k \in z$

So, $\omega^n + (\omega^2)^n = \omega^{3k+1} + \omega^{6k+2}$

$= (\omega^3)^k w + (\omega^2)^{2k}\omega^2 = \omega + \omega^2 = -1$

87.(b) Let α and β be the roots of the equation. Then $\alpha + \beta = p$, $\alpha\beta = q$

As $\alpha + \beta = m(\alpha - \beta)$

$\Rightarrow (\alpha + \beta)^2 = m^2(\alpha - \beta)^2$

$\Rightarrow (\alpha + \beta)^2 = m^2\{(\alpha + \beta)^2 - 4\alpha\beta\}$

$\Rightarrow p^2 = m^2(p^2 - 4q)$

$\Rightarrow p^2(1 - m^2) = -4m^2q$

88.(c) $\frac{1^3 + 2^3 + 3^3 + ... + 12^3}{1^2 + 2^2 + 3^2 + ... + 12^2} = \frac{3\ 12(12+1)}{2.(2.12+1)} = \frac{234}{25}$

$\left[\frac{s_n^{\ 3}}{s_n^{\ 2}} = \frac{3n(n+1)}{2(2n+1)}\right]$

89.(a) $f\left(\frac{x}{y}\right) = \cos\log\frac{x}{y} = \cos(\log x - \log y)$

$fxy = \cos\log xy = \cos(\log x + \log y)$

$f(xy)+f(x, y)=\cos(\log x+\log y)+\cos(\log x-\log y)$

$= 2\cos\log x - \cos\log y = 2f(x)f(y)$

Thus, $f(x)\ f(y) - \frac{1}{2}[2f(x)\ f(y)] = 0$

90.(d) Given $|\vec{a} + \vec{b}| = 1$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b} = 1$

$\Rightarrow 1 + 1 + 2\vec{a}.\vec{b} = 1$

$\Rightarrow 2\vec{a}.\vec{b} = -1$

So, $|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 = -4\vec{a}.\vec{b} = 1+1+1 = 3$

$\Rightarrow |\vec{a} - \vec{b}| = \sqrt{3}$

91.(a) Here, n = 6 in which A & N are repeated 2 times. So no of arrangements $= \frac{6!}{2!\ 3!} = 60$

When 2N's are together, no. of arrangements $= \frac{5!}{3!} = \frac{120}{6} = 20$

$\therefore$ No. of arrangements in which 2N's are never together

$= 60 - 20 = 40$

92.(a) Making homogenous curve with the help of line $2x + y = 1$, we have $3x^2 + 4xy - 4x(2x + y) + (2x + y)^2 = 0$

i.e. $-x^2 + 4xy + y^2 = 0$

Here coeff. of x^2 + coeff. of $y^2 = 0$.

So angle is 90°

93.(a) Here dr's of OA are a – 0, b – 0, c – 0

i.e. a, b, c

So the equation of required plane is $a(x - a) + b(y - b) + c(z - c) = 0$

94.(c) Given hyperbolas are rectangular hyperbola.

So $e = 2$, $e_1 = 2$

Then $e^2 + e_1^{\ 2} = 4 + 4 = 8$

95.(a) Here $\frac{dx}{dt} = 3$ m/s, $\frac{dy}{dt} = ?$

From similar Δs, we have

$\frac{2}{5} = \frac{y}{x+y} \Rightarrow 2x + 2y = 5y$

$\Rightarrow 3y = 2x$

i.e. $3\frac{dy}{dx} = 2.\frac{dx}{dt} \Rightarrow \frac{dy}{dt} = 2$ m/s

96.(b) Solving $y = x^3$ and $y = x$, we have

$x^3 - x = 0 \Rightarrow x = 0, -1, 1$

$\therefore$ Required area $= 2\int_0^1 (x - x^3)\,dx$ (By symmetry)

$= 2\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1$

$= 2\left[\frac{1}{2} - \frac{1}{4}\right] = 2.\frac{1}{4} = \frac{1}{2}$ sq. units

97.(c) 98.(b) 99.(c) 100.(d)

…Best of Luck…