Himalaya College of Engineering Chyasal, Lalitpur Tel: 01-5440555, 5454227, 9851063670

2081-4-12 Hints & Solution

Section – I

1.(c) From De Morgan's law, $\sim(p \vee q) \equiv \sim p \wedge \sim q$.

2.(c)

3.(b) $Z^n + Z^{-n} = (\cos\theta + i\sin\theta)^n + (\cos\theta + i\sin\theta)^{-n}$
$= \cos n\theta + i \sin n\theta + \cos(-n\theta) + i \sin(-n\theta)$
$= \cos n\theta + i \sin n\theta + \cos n\theta - i \sin n\theta$
$= 2\cos n\theta$

4.(a) $(1 + 2x + x^2)^{15} = ((1 + x)^2)^{15} = (1 + x)^{30}$
No. of terms = n + 1 = 30 + 1 = 31

5.(b) $t_n = s_n - s_{n-1}$
$\therefore \quad t_{10} = s_{10} - s_9 = (10^3 - 100) - (9^3 - 100) = 271$

6.(b) $\sec^2\theta + \text{cosec}^2\theta$
$= 1 + \tan^2\theta + 1 + \cot^2\theta$
$= 2 + (\tan\theta - \cot\theta)^2 + 2\tan\theta \cot\theta$
$= 4 + (\tan\theta - \cot\theta)^2 \geq 4$
Min. value = 4

7.(c) $\sec^2(\tan^{-1} 2) + \text{cosec}^2(\cot^{-1} 3)$
$= 1 + \tan^2(\tan^{-1} 2) + 1 + \cot^2(\cot^{-1} 3)$
$= 1 + 2^2 + 1 + 3^2 = 15$

8.(c) $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0 \Rightarrow abc + 2 = a + b + c$

9.(a) Formula

10.(a)

11.(c)

12.(a) $y = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \ldots$
$y = e^x$
$\dfrac{dy}{dx} = e^x$
$\therefore \quad \dfrac{dy}{dx} = y$

13.(a) $v = \dfrac{ds}{dt} = 6t - 8$
The body will be stopped when v = 0
i.e. $\quad 6t - 8 = 0$
$\Rightarrow \quad t = \dfrac{4}{3}$ sec

14.(b) Put $x^2 = t \Rightarrow dt = 2xdx$
$I = \dfrac{1}{2} \int \cos t \, dt = \dfrac{1}{2} \sin t + c = \dfrac{1}{2} \sin x^2 + c$

15.(b) $y = (C_1 + C_2) \sin(x + C_3) - C_4 e^x . e^{C_5}$
$y = A \sin(x + C_3) - Be^x$
Where $A = C_1 + C_2$ & $B = C_4 e^{C_5}$
Order = no. of arbitrary constants = 3
(A, B & C_3)

16.(d) $m_1 + m_2 = 4m_1m_2$
or, $\quad -\dfrac{2C}{7} = 4\left(-\dfrac{1}{7}\right)$
$\Rightarrow \quad C = 2$

17.(d) g = 5, c = 9
Length of intercept on x-axis $= 2\sqrt{g^2 - c}$
$= 2\sqrt{5^2 - 9} = 8$

18.(c)

19.(d) Distance from y-axis $= \sqrt{x^2 + z^2} = \sqrt{3^2 + 5^2} = \sqrt{34}$

20.(c) Since the events are independent, P(A and B) = $P(A \cap B) = P(A).P(B)$
$= \dfrac{2}{3} \times \dfrac{1}{3} = \dfrac{2}{9}$

21.(d) $\vec{A}.\vec{B} = |\vec{A} \times \vec{B}|$
or, $\quad AB\cos\theta = AB\sin\theta$
or, $\quad \tan\theta = 1 = \tan 45°$
$\theta = 45°$
$R = \sqrt{A^2 + 2AB\cos 45° + B^2} = \sqrt{A^2 + \sqrt{2}\,AB + B^2}$

22.(d) Impulse = change in momentum
$= mv - (-mu)$
$= mv + mu$
$= 0.1\,(20 + 30) = 5$ NS

23.(b) $\dfrac{Ir}{Id} = \dfrac{mr^2}{\frac{1}{2} mr^2} = 2:1$

24.(c) Internal energy = KE
KE is function of temp.

25.(d) No work is done in isochoric process.

26.(a) Ultrasonic, infrasonic and audible are classified with frequency but speed remain same in medium.

27.(c) $V = \dfrac{W}{Q} = \dfrac{100}{-5} = -20V$

28.(c) $V = E - Ir = E - \dfrac{E}{R + r} . r$
$= 2 - \dfrac{2}{3.9 + 0.1} \times 0.1$
$= 1.95$ V

29.(b) $B_H = B\cos\delta$
or, $\quad B_0 = B \times \cos 45°$
or, $\quad B = \sqrt{2}\, B_0$

30.(a) At resonance
$f_0 = \dfrac{1}{2\pi\sqrt{LC}}$
i.e. $\quad LC = \text{constant}$
or, $\quad LC = \dfrac{L}{2} \times C'$
or, $\quad C' = 2C$

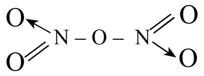
31.(d) $\beta = \dfrac{D\lambda}{d}$
$\beta' = \dfrac{2D\lambda}{\frac{d}{2}} = 4\beta$

32.(a) $\delta = (\mu - 1)\, A$
$\mu = A + \dfrac{B}{\lambda^2}$ so μ decreases
If λ increases and δ decreases
If μ is least

33.(d) $qV = \dfrac{1}{2} mv^2$
or, $\quad v = \sqrt{\dfrac{2qV}{m}}$
$\therefore \quad \dfrac{v_{He}}{v_H} = \sqrt{\dfrac{2 \times 2eV \times m}{4m \times 2eV}}$
$= \dfrac{1}{\sqrt{2}}$

34.(b) **For diode**
P = IV
or, $\quad I = \dfrac{100 \times 10^{-3}}{0.5} = 0.2A$
Now, $I = \dfrac{1.5 - 0.5}{R}$ $\quad$ or, $r = \dfrac{1}{0.2} = 5\Omega$

35.(b)



Thus covalent & co-ordinate.

36.(d) $\overset{0}{H_2} + Ca \longrightarrow Ca\overset{-1}{H_2}$

Reduced

⇓

Oxidizing agent

37.(a) For 4d

h = 4   e = 2

m = –2 to +2

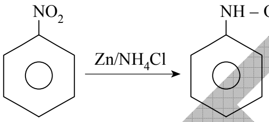
$s = \pm\frac{1}{2}$

38.(b) $\frac{0.53}{53} = \frac{100 \times 0.1}{1000}$

$\frac{1}{100}$ gm eq $= \frac{1}{100}$ gm eq

Neutral

39.(c)

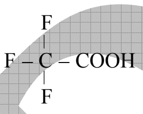


40.(a)

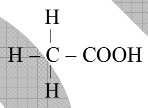
41.(d)

42.(b) –OH group is activating group. So it favour's electrophilic substitution reaction.

43.(a)



$$F-\overset{F}{\underset{F}{C}}-COOH$$



$$H-\overset{H}{\underset{H}{C}}-COOH$$

–I effect of F > H

So trifluoroacetic acid stronger than acetic acid.

44.(d) Iodoform test

45.(c)

46.(c) $Fe_2O_3 + CO \rightarrow 2Fe + CO_2$

47.(b) $KBr + \underset{\text{conc.}}{H_2SO_4} \rightarrow Br_2 + SO_2$

48.(a)

49.c	**50.a**	**51.c**	**52.c**	**53.d**	**54.d**
55.b	**56.b**	**57.b**	**58.d**	**59.a**	**60.c**

Section – II

61.(b) $y = \sin^{-1}\left[\log_3\left(\frac{x}{3}\right)\right]$

$\Rightarrow\quad -1 \le \log_3\left(\frac{x}{3}\right) \le 1$

$\Rightarrow\quad 3^{-1} \le \frac{x}{3} \le 3^1$

$\Rightarrow\quad \frac{1}{3} \le \frac{x}{3} \le 3$

$\Rightarrow\quad 1 \le x \le 9$

$\Rightarrow\quad x \in [1, 9]$

62.(c) $(a + b + c)(b + c - a) = 3bc$

$\Rightarrow\quad (b + c)^2 - a^2 = 3bc$

$\Rightarrow\quad b^2 + c^2 - a^2 = bc$

$\Rightarrow\quad \frac{b^2 + c^2 - a^2}{2bc} = \frac{1}{2}$

$\Rightarrow\quad \cos A = \cos 60°$

$\Rightarrow\quad A = 60°$

63.(d) $|\vec{a} + \vec{b}| = 1$

or, $(\vec{a} + \vec{b})^2 = 1$

or, $a^2 + 2\vec{a}.\vec{b} + b^2 = 1$

or, $1 + 2\vec{a}.\vec{b} + 1 = 1$ $[\because |\vec{a}| = |\vec{b}| = 1]$

$\therefore\quad 2\vec{a}.\vec{b} = -1$

Now, $|\vec{a} - \vec{b}|^2 = (\vec{a} - \vec{b})^2$

$= a^2 - 2\vec{a}.\vec{b} + b^2$

$= 1 - (-1) + 1 = 3$

$|\vec{a} - \vec{b}| = \sqrt{3}$

64.(d) Taking common a, b, c from R_1, R_2 and R_3

$$abc\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix}$$

Again, taking common a, b, c from C_1, C_2 & C_3

$$(abc)\,(abc)\begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

Expanding along R_1

$a^2b^2c^2.4 = 4a^2b^2c^2$

65.(c) For even numbers, we must have 0, 2, 4 or 6 in unit's place. Also, the first digit cannot be 0. We have two cases.

Case I: Last digit is 0

Required no. of ways = 4 × 5 × 6 × 1

= 120

Case II: Last digit is 2, 4, or 6

Required no. of ways = 5 × 5 × 4 × 3 = 300

Total no. of 4 digit even numbers = 120 + 300 = 420

66.(b) Since ratio of the roots are equal, $\frac{\alpha}{\beta} = \frac{\alpha_1}{\beta_1}$

or, $\frac{\alpha + \beta}{\alpha - \beta} = \frac{\alpha_1 + \beta_1}{\alpha_1 - \beta_1}$ (Componendo & dividendo)

or, $\frac{(\alpha + \beta)^2}{(\alpha_1 + \beta_1)^2} = \frac{(\alpha + \beta)^2 - 4\alpha\beta}{(\alpha_1 + \beta_1)^2 - 4\alpha_1\beta_1}$

or, $\frac{p^2}{l^2} = \frac{p^2 - 4q}{l^2 - 4m} \Rightarrow p^2m = l^2q$

67.(d) General term $(t_n) = \frac{\frac{1(2^n - 1)}{2 - 1}}{n!}$

$[\because 1 + 2 + 2^2 + 2^3 + \ldots$ is a G.S.]

$= \frac{2^n - 1}{n!} = \frac{2^n}{n!} - \frac{1}{n!}$

$S_\infty = \sum_{n=1}^{\infty}\left(\frac{2^n}{n!} - \frac{1}{n!}\right) = (e^2 - 1) - (e - 1) = e^2 - e$

68.(a) Let the coordinates of the foot of the perpendicular be (x_2, y_2)

Then, $\frac{x_2 - x_1}{a} = \frac{y_2 - y_1}{b} = -\frac{ax_1 + by_1 + c}{a^2 + b^2}$

i.e. $\frac{x_2 - 2}{1} = \frac{y_2 - 4}{1} = -\frac{(2 + 4 - 1)}{1 + 1}$

From 1st and 3rd, $x_2 = -\frac{1}{2}$

From 2^{nd} and 3^{rd}, $y_2 = \frac{3}{2}$

Required point = $\left(-\frac{1}{2}, \frac{3}{2}\right)$

69.(d) The eq^n of plane through the point (2, –3, 1) is $a(x - 2) + b(y + 3) + c(z - 1) = 0$ ... (i)
D.r's of the line joining the points (3, 4, –1) and (2, –1, 5) are
(3 – 2), (4 + 1), (–1 –5) i.e. 1, 5, –6
Required eq^n of plane is
$1(x - 2) + 5(y + 3) - 6(z - 1) = 0$
$\therefore \quad x + 5y - 6z + 19 = 0$

70.(a) (1, –2) lies on $y^2 = 4ax$
i.e. $4 = 4a.1 \Rightarrow a = 1$
The tangent at the point $(x_1, y_1) = (1, -2)$ is
$y.y_1 = 2a(x + x_1)$
$y.(-2) = 2.1(x + 1)$
$x + y + 1 = 0$

71.(c) $\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$

$$= \sqrt{\frac{\frac{n(n+1)(2n+1)}{6}}{n} - \left(\frac{\frac{n(n+1)}{2}}{n}\right)^2}$$

$$= \sqrt{\frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}}$$

$$= \sqrt{\frac{(n+1)(n-1)}{12}} = \sqrt{\frac{n^2-1}{12}}$$

72.(b) Let $y = \lim_{x \to 0} x^x$

Then $l\text{ny} = \lim_{x \to 0} x\,l\text{nx} = \lim_{x \to 0} \frac{l\text{nx}}{\frac{1}{x}}$

$$= \lim_{x \to 0} \frac{\frac{1}{x}}{-\frac{1}{x^2}}$$ [Using L' Hospital's rule]

$= 0 \quad \therefore \; l\text{ny} = 0 \quad \Rightarrow \; y = e^0 = 1$

73.(d) $y = \tan^{-1}\left(\frac{\cos x + \sin x}{\cos x - \sin x}\right)$

$$= \tan^{-1}\left(\frac{1 + \tan x}{1 - \tan x}\right)$$

$$= \tan^{-1}\left(\frac{\tan\frac{\pi}{4} + \tan x}{1 - \tan\frac{\pi}{4}.\tan x}\right)$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4} + x\right)\right) = \frac{\pi}{4} + x$$

$\frac{dy}{dx} = 1$

74.(a) $\int_0^{\sqrt{2}} [x^2]\,dx = \int_0^1 [x^2]\,dx + \int_1^{\sqrt{2}} [x^2]\,dx$

$$= \int_0^1 0\,dx - \int_1^{\sqrt{2}} dx$$

$$= [x]_1^{\sqrt{2}} = \sqrt{2} - 1$$

75.(c) $y^2 = x$ and $y = |x|$
$\Rightarrow \quad x^2 = x$
$\Rightarrow \quad x = 0, 1$

Required area = $\int_0^1 (\sqrt{x} - x)\,dx = \left[\frac{2}{3}x^{3/2} - \frac{x^2}{2}\right]_0^1$

$$= \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$$

76.(d) **1^{st} case**
$v^2 = 0^2 + 2gh$
or, $\quad v = \sqrt{2 \times 10 \times 50} = \sqrt{1000}$
When open
$v^2 = u^2 - 2ah'$
or, $\quad h' = \frac{1000 - 3^2}{2 \times 2} = 247.75$ m
Height = 50 + 247.75
= 298 m

77.(b) $\frac{1}{2}mv^2 = \frac{1}{2}kx^2$

or, $\quad x = \sqrt{\frac{mv^2}{K}} = \sqrt{\frac{0.5 \times 1.5^2}{50}} = 0.15$ m

78.(a) Change in wt = change in upthrust
or, $\quad mg = (l \times b \times \Delta h)\sigma g$
or, $\quad m = 3 \times 2 \times 0.01 \times 1000 = 60$ kg

79.(a) No of moles (n) = $\frac{5.6}{22.4} = \frac{1}{4}$

$T_2V_2^{\gamma-1} = T_1V_1^{\gamma-1}$

or, $\quad T_2 = T_1\left(\frac{5.6}{0.7}\right)^{\frac{5}{3-1}}$

$= T_1\,(8)^{2/3} = 4T_1$

$W = \frac{nR[T_1 - T_2]}{\gamma - 1}$

$= \frac{1}{4}R\,\frac{[T_1 - 4T_1]}{5/3 - 1} = -\frac{1}{4}R \times 3T_1 \times \frac{3}{2}$

$= -\frac{9RT_1}{8} = \frac{9RT_1}{8}$

80.(d) In parallel

$$\frac{Q}{t} = \frac{K2Ad\theta}{l} = q_1L_f \text{ ... (i)}$$

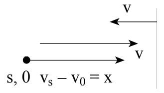
In series

$$\frac{Q}{t} = \frac{KAd\theta}{2l} = q_2L_f \text{ ...(ii)}$$

(ii) ÷ (i)

$\frac{q_2}{q_1} = \frac{KAd\theta}{2l} \times \frac{h}{K.2Ad\theta} = \frac{1}{4}$

81.(c)



$f' = \frac{v + v_0}{v - v_s} \times f$

or, $2f = \frac{v + x}{v - x} f$

or, $2v - 2x = v + x$

or, $x = \frac{v}{3}$

82.(a) $E = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$

$= -\{6y\hat{i} + (6x - 1 + 2z)\hat{j} + 2y\hat{k}\}$

$= \{6\hat{i} + (6 - 1 + 2 \times 0)\hat{j} + 2\hat{k}\}$

$= -(6\hat{i} + 5\hat{j} + 2\hat{k})$

83.(a) $Q = at - bt^2$

$I = \frac{dQ}{dt} = a - 2bt$

When $I = 0$, $a - 2bt = 0$ or, $t = \frac{a}{2b}$

$E = \int_0^t Pdt = \int_0^t I^2Rdt$

$= \int_0^{a/2b} (a - 2bt)^2\, Rdt = \int_0^{a/2b} (a^2 - 4abt + 4b^2t^2)\, Rdt$

$= \int_0^{a/2b} a^2Rdt - \int_0^{a/2b} 4abRtdt + 4b^2R \int_0^{a/2b} t^2dt$

$= a^2R(t)_0^{a/2b} - 4abR\left(\frac{t^2}{2}\right)_0^{a/2b} + 4b^2R\left(\frac{t^3}{3}\right)_0^{a/2b}$

$= a^2R^2 \times \frac{a}{2b} - \frac{4abR}{2}\left(\frac{a}{2b}\right)^2 + \frac{4b^2R}{3}\left(\frac{a}{2b}\right)^3$

$= \frac{a^3R}{2b} - \frac{2a^3R}{4b} + \frac{4b^2R \times a^3}{24b^3}$

$= \frac{a^3R}{2b} - \frac{a^2R}{2b} + \frac{a^3R}{6b} = \frac{a^3R}{6R}$

84.(b) $Q = \frac{d\phi}{R} = \frac{ANdB}{R} = \frac{\pi r^2N \times \mu_0 n(I - 0)}{R}$

$= \frac{\pi \times 0.01^2 \times 100 \times 4\pi \times 10^{-7} \times 2 \times 10^4 \times 4}{10\pi^2}$

$= 32 \times 10^{-6}C = 32\ \mu c$

85.(c) At λ path difference $\phi = 2\pi$

$I_{max} = 4I$

or, $I = \frac{K}{4}$

If path difference is $\frac{\lambda}{4}$ then $\phi = \frac{\pi}{2}$

$I_R = I_1 + 2\sqrt{I_1I_2}\cos\phi + I_2$

$= 2I = 2\frac{K}{4} = \frac{K}{2}$

86.(c) For lyman series

$\frac{1}{\lambda_L} = R\left[\frac{1}{1} - \frac{1}{2^2}\right]$

or, $\lambda_L = \frac{4}{3R}$ ... (i)

For Balmer series

$\frac{1}{\lambda_B} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$

∴ $\lambda_B = \frac{36}{5R}$ ... (2)

(1) ÷ (2)

$\frac{\lambda_L}{\lambda_B} = \frac{4}{3R} \times \frac{5R}{36} = \frac{5}{27}$

87.(c) For air

$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

$\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{20} \times \frac{1}{(1.6 - 1)} = \frac{1}{12}$ ...(1)

In liquid

$\frac{1}{f_l} = ({}_l\mu_g - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

$= \left(\frac{{}_a\mu_g}{{}_a\mu_l} - 1\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

$= \left(\frac{1.6}{1.3} - 1\right) \times \frac{1}{12}$

$= \frac{0.3}{1.3} \times \frac{1}{12} = \frac{1}{52}$

∴ $f_l = 52$ cm convex

88.(d) $A = \lambda N_0 = \frac{0.693}{T_{1/2}} \times \frac{N_A}{226}$

$= \frac{0.693 \times 6.023 \times 10^{23}}{1620 \times 365 \times 24 \times 3600 \times 226}$

$= 3.6 \times 10^{10}$ dis/sec

89.(d) For spontaneous $\Delta G = -ve$ in $\Delta G = \Delta H - T\Delta S$

Since for all $\Delta G = +ve$

So none of them

90.(c) Half means 50%

$+\frac{1}{2} = 40$

$\frac{3}{4}^{th}$ means $2 \times t_{1/2}$

So, 2 × 40 = 80 minute

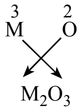
91.(c) $\frac{\text{wt of oxygen}}{\text{eq. wt of oxygen}} = \frac{\text{wt of metal}}{\text{eq. wt of metal}}$

$\frac{47}{8} = \frac{53}{X}$

$X = 9.02$

Relative atomic mass = 27

Valency $= \frac{27}{9} = 3$



$\overset{3}{M}\ \ \overset{2}{O} \rightarrow M_2O_3$

92.(c)

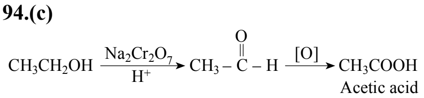
93.(b) Here,

336ml = 1.2 gm

$22400\ ml = \frac{1.2}{336} \times 22400$

$= 80$ g of gas

94.(c)



$CH_3CH_2OH \xrightarrow[H^+]{Na_2Cr_2O_7} CH_3 - \overset{\overset{\large O}{\|}}{C} - H \xrightarrow{[O]} \underset{\text{Acetic acid}}{CH_3COOH}$

95.(b)

96.(b)

97.b **98.b** **99.d** **100.a**

…The End…