PEA's

TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

B.E. Model Entrance Exam

2079

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Hints and Solutions

Section – I

1.(b) $\tan\theta = \frac{u_y}{u_x}$ or, $\theta = \tan^{-1}\left(\frac{b}{a}\right)$

2.(b) $f = \frac{1}{T} = \frac{1}{2 \times 0.25} = 2$ Hz

3.(c) Smaller it the least count of device, more accurate and precise is the measurement. Measurement 1.000 cm is done by device of least count 0.001 cm

4.(b) $a = \sqrt{a_r^2 + a_t^2}$

or, $a = \sqrt{\left(\frac{v^2}{r}\right)^2 + a_t^2}$

or, $a = \sqrt{\left(\frac{20^2}{100}\right)^2 + 3^2}$

or, $a = 5\ m/s^2$

5.(a)

6.(c)

7.(d) $R.H. = \frac{\text{S.V.P. at dew point}}{\text{S.V.P. at room temperature}} \times 100\%$

$= \frac{x}{x} \times 100\%$

($\because$ dew point = room temperature)

$= 100\%$

8.(a) $\phi = \frac{1}{\varepsilon_0} q_{net} = 0$ ($\because q_{net} = 0$)

9.(a) Persistence of hearing (t) $= \frac{1}{10}$ sec

$\therefore$ $2d = V \times t$

or, $d = \frac{V \times t}{2}$

or, $d = \frac{V}{2} \times \frac{1}{10} = \frac{V}{20}$

10.(d) $\beta = \frac{\lambda D}{d}$, i.e. $\beta \propto \lambda$

When frequency is doubled, wavelength is halfed, so fringe width is halved.

11.(a) $I = \frac{\varepsilon}{r}$

or, $r = \frac{1.5}{3}$ or, $r = 0.5\ \Omega$

12.(d) $\phi = LI$

$= 8 \times 10^{-3} \times 12 \times 10^{-3}$

$= 96 \times 10^{-6}$ Wb

13.(d) γ-ray is originated from nuclear transition.

14.(c) Energy stored = Energy density × volume

$= \frac{1}{2}\varepsilon_0 E^2 \times V$

$= \frac{1}{2} \times 8.854 \times 10^{-12} \times 120^2 \times 1$

$= 6.37 \times 10^{-8}$ J

15.(a) $v = \frac{E}{B} = \frac{1.50 \times 10^3}{0.350} = 4.28 \times 10^3$ m/s

16.(a) Since, diode is reverse biased, reading of A_1 is OA. For A_2,

$I = \frac{E}{5} = \frac{5}{5} = 1A$

17.(b) $W = \frac{1}{2} \times F \times \Delta l$, F = thermal force

Δl = extension

or, $W = \frac{1}{2} \times YA\alpha\theta \times l\alpha\theta$

or, $W = \frac{1}{2} YAl\alpha^2\theta^2$

18.(a) Bornit is Cu_3FeS_3

19.(b)

20.(a)

21.(a) $SO_2 + 2H_2O \rightarrow H_2SO_4 + 2[H]$

22.(c)

23.(a)

24.(a)

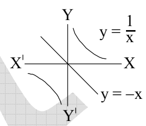
25.(d)

26.(d)

27.(b)

28.(c)

29.(c)



Plotting the graphs

These graphs do not intersect. So $A \cap B = \phi$

30.(b) We have A. Adj.A = |A| I, So, $|A| \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$= 10\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow |A| = 10$

31.(c) $|z| = |3 + 4i| = \sqrt{3^2 + 4^2} = 5$ and $|w| = |4 - 3i|$

$= \sqrt{4^2 + (-3)^2} = 5$

So, $|z| = |w|$

32.(d) $\lim_{x \to 0} \frac{\log_a(1 + x)}{x} = \lim_{x \to 0} \log_a e . \frac{\log_e(1 + x)}{x}$

$= \log_a e . 1 = \log_a e$

33.(a) $y = 1 + x + x^2 + \ldots$ to $\infty = \frac{1}{1 - x}$

$\Rightarrow 1 - x = \frac{1}{y} \Rightarrow x = 1 - \frac{1}{y} = \frac{y - 1}{y}$

34.(c) Put $y = \sqrt{x}$ $\therefore dy = \frac{1}{2\sqrt{x}} dx$

i.e. $2dy = \frac{1}{\sqrt{x}} dx$. Then

$\int \frac{2\sqrt{x}}{\sqrt{x}} dx = 2\int e^y dy = 2e^y + c = 2e^{\sqrt{x}} + c$

35.(a) Replacing y by –y, we have $(-y)^2 = 4ax$

i.e. $y^2 = 4ax$. So it is symmetric about x-axis.

36.(d) $(1 + 3x + 3x^2 + x^3)^{15} = (1 + x)^{45}$

So, coeff. of x^{10} is c(45, 10)

37.(d) As origin is the centre of the circle $x^2 + y^2 = 25$. So infinitely many normals are possible.

38.(b) Equation of ellipse is $3x^2 + 4y^2 = 12$

$\Rightarrow \frac{x^2}{4} + \frac{y^2}{3} = 1$

So, $e = \sqrt{1 - \frac{3}{4}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

39.(b) $|\vec{a} \times \vec{b}| = (\vec{a}.\vec{b}) \Rightarrow |\vec{a}|\,|\vec{b}| \sin\theta = |\vec{a}|\,|\vec{b}| \cos\theta$

$\Rightarrow \tan\theta = 1 \;\Rightarrow \theta = 45°$

40.(d) cosecA (sinB cosC + cosB sincC)
= cosecA.sin(B + C)
= cosecA.sin(π – A)
= cosecA.sinA = 1

41.(d) $\sin \text{arc} \cos t = \sin \cos^{-1} t = \sin \sin^{-1}\sqrt{1-t^2}$
$= \sqrt{1-t^2} = \cos \cos^{-1}\sqrt{1-t^2}$
$= \cos \sin^{-1} t$
= cos(arc sint)

42.(c) For minimum value, we must have,
$4x + 20 = 0 \Rightarrow x = -5$

43.(c) Given $\cos x + \cos^2 x = 1$

$\Rightarrow \cos x = 1 - \cos^2 x = \sin^2 x$

So $\sin^2 x + \sin^4 x = \sin^2 x + (\sin^2 x)^2 = \cos x + \cos^2 x = 1$

44.(b) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 1 - \cos^2\alpha + 1 - \cos^2\beta + 1 - \cos^2\gamma$
$= 3 - (\cos^2\alpha + \cos^2\beta + \cos^2\gamma) = 3 - 1 = 2$

45.(a) Comparing with $ax^2 + 2hxy + by^2 = 0$, we get a = 1, h = –3, b = 9
Here $h^2 = (-3)^2 = 9 = 1.9 = ab$
So angle = 0°

46.(b) As both roots are zero, so constant term = 0, coeff of x = 0
$\Rightarrow \lambda = 0, k = 2$

47.(c) $y = \sin^{-1}\frac{2\sqrt{x}}{\sqrt{x}+1} + \sec^{-1}\frac{\sqrt{x}+1}{2\sqrt{x}} = \sin^{-1}\frac{2\sqrt{x}}{\sqrt{x}+1} + \cos^{-1}\frac{2\sqrt{x}}{\sqrt{x}+1} = \frac{\pi}{2}$

So, $\frac{dy}{dx} = 0$

48.(d) Given, $2y = 2 - x^2$, on differentiation, we have $\frac{dy}{dx} = -x$

At x = 1, $\frac{dy}{dx} = -1 \Rightarrow \tan\theta = -1 \Rightarrow \theta = 135°$

49.(a)	50.(a)	51.(c)	52.(d)	53.(a)	54.(c)
55.(c)	56.(d)	57.(a)	58.(b)	59.(c)	60.(b)

Section – II

61.(b) T – mg = ma

or, $a = \frac{T - mg}{m}$

or, $a = \frac{50{,}000 - 5000 \times 9.8}{5000}$

or, $a = 0.2 \text{ m/s}^2$

Then, $h = \frac{1}{2}at^2 = \frac{1}{2} \times 0.2 \times 10^2 = 10$

62.(c) ω = a – bt ….(i)

$\therefore \alpha = \frac{d\omega}{dt}$

or, $\alpha = -b$ rad/s^2 (retardation which is uniform)
When t = 0, then
ω = a rad/s
or, ω_0 = a rad/s
Then, $\omega = \omega_0^2 + 2\alpha\theta$
or, $0 = a^2 - 2 \times b \times \theta$
(∵ ω = 0 as it come to rest)

or, $\theta = \frac{a^2}{2b}$

63.(b) Velocity of efflux is
$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 5} = 10$ m/s
Then volume per second is
$V = Av = 1 \times 10^{-4} \times 10 = 10^{-3}$ m^3/s

64.(a) Work-done = change in K.E.

or, $\int_{x=20\,m}^{x=30m} F dx = K.E._f - K.E_i$

or, $\int_{20}^{30} (-0.1x)\, dx = K.E_f - \frac{1}{2} mu^2$

or, $-0.1\left[\frac{x^2}{2}\right]_{20}^{30} = K.E_f - \frac{1}{2} \times 10 \times 10^2$

or, $-\frac{0.1}{2}[30^2 - 20^2] = K.E_f - 500$

or, $K.E_f = 500 - 25$
or, $K.E_f = 475$ J

65.(d) $\mu = \frac{\text{Real depth}}{\text{Apparent depth}}$

∴ Real depth = 1.5 × 7 = 10.5 cm

66.(c) 50% of $\frac{1}{2}mv^2 = ms\Delta\theta + mL_f$

or, $\frac{1}{4}v^2 = s\Delta\theta + L_f$

or, $\frac{1}{4}v^2 = 0.03 \times 4200 \times (330 - 20) + 6 \times 4200$

or, v = 506.9 m/s ≈ 507 m/s

67.(d) $C_{rms} = \sqrt{\frac{3RT}{M}}$

i.e. $C_{rms} \propto \sqrt{\frac{T}{M}}$

$\therefore \frac{(C_{rms})_{H_2}}{(C_{rms})_{N_2}} = \sqrt{\frac{T_{H_2}}{M_{H_2}} \times \frac{M_{N_2}}{T_{N_2}}}$

or, $7 = \sqrt{\frac{T_{H_2}}{2} \times \frac{28}{300}}$

or, $T_{H_2} = \frac{49 \times 2 \times 300}{28}$

or, $T_{H_2} = 1050$ K

68.(a) $P = I_{rms} V_{rms} \cos\phi$; ϕ = phase difference between current & voltage

$$= \frac{I_0}{\sqrt{2}} \times \frac{V_0}{\sqrt{2}} \times \cos\frac{\pi}{3} = \frac{2 \times 5}{2} \times \frac{1}{2} = 2.5 \text{ watts}$$

69.(d) $B = \mu_0 nI$

$= 4\pi \times 10^{-7} \times 12300 \times 3.60$

$= 0.055$ T

$\therefore$ $F_m = Bev\sin\theta$

$= Be \times 0.0187C \times \sin 90°$

$= 0.055 \times 1.6 \times 10^{-19} \times 0.0187 \times 3 \times 10^8$

$= 4.94 \times 10^{-14}$ N

70.(b) $\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{2C} + \frac{1}{C}$

or, $\frac{1}{C_{eq}} = \frac{5}{2C}$

or, $C_{eq} = \frac{2C}{5}$

or, $C_{eq} = \frac{2 \times 4}{5}$

or, $C_{eq} = \frac{8}{5}\,\mu F$

or, $C_{eq} = \frac{8}{5} \times 10^{-6}$ F

Then, $E = \frac{1}{2} C_{eq} V^2$

$= \frac{1}{2} \times \frac{8}{5} \times 10^{-6} \times 15^2 = 1.8 \times 10^{-4}$ J

71.(c) $\frac{N}{N_0} = \left(\frac{1}{e}\right)^{t/T}$

or, $\frac{N}{N_0} = \left(\frac{1}{e}\right)^{3T/T}$

or, $\frac{N}{N_0} = \frac{1}{e^3} \times 100\%$

$= 4.97\%$

72.(a) $K.E_p$ = rest-mass of electron

or, $K.E_p = m_0c^2$

or, $K.E_p = 9.1 \times 10^{-31} \times (3 \times 10^8)^2$

or, $K.E_p = 8.2 \times 10^{-14}$ J

Then, $\lambda_p = \frac{h}{\sqrt{2m_pK.E_p}}$

$= \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 1.66 \times 10^{-27} \times 8.2 \times 10^{-14}}}$

$= 4.012 \times 10^{-14}$ m

$= 40.12 \times 10^{-15}$ m $= 40.12$ fm

73.(b) The output power delivered by light source is

$P_{out} = \frac{n}{t}\frac{hc}{\lambda}$ or, $P_{out} = \frac{n}{t} \times \frac{hc}{\lambda}$

or, $P_{out} = \frac{2.3 \times 10^{20} \times 6.62 \times 10^{-34} \times 3 \times 10^8}{600 \times 10^{-9}}$

or, $P_{out} = 76.13$ watt

$\therefore$ $\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{76.13}{100} \times 100\% = 76.13\%$

74.(a) $V_s = \omega_r$

or, $V_s = 2\pi fr$

or, $V_s = 2\pi \times \frac{400}{60} \times 1.2$

or, $V_s = 50.26$ m/s

Then, $f' = \frac{V}{v + v_s} \times f$

$= \frac{340}{340 + 50.26} \times 500 = 435.6$ Hz

75.(b) According to Simon's (n + l) rule the highest energy is in n = 3 and l 2 (3 + 2 = 5]

76.(a) $CaOCl_2 + 2HCl \rightarrow CaCl_2 + H_2O + Cl_2$

77.(c) Conc^n $NHO_3 \rightarrow H_2O + 2NO_2 + [O]$

$I_2 + 5[O] \rightarrow I_2O_5$

$I_2O_5 + H_2O \rightarrow 2HIO_3$

$HIO_3 \rightarrow HOIO_2$

78.(c)

79.(c) $Al^{3+} + 3e^- \rightarrow Al \; \frac{\text{At. mass}}{3} \Rightarrow E_{Al}$

$Cu^{2+} + 2e^- \rightarrow Cu \; \frac{\text{At mass}}{2} \Rightarrow E_{Cu}$

$Na^+ + 1e^- \rightarrow Na \; \frac{\text{At. mass}}{1} \Rightarrow E_{Na}$

80.(d) $N_{mix} = \frac{5 \times 1 + 20 \times 0.5 + 30 \times 0.33}{5 + 20 + 30}$

$= \frac{5 + 10 + 10}{55} = 0.436$ N

$0.436 \times 55 = 1000 \times N$

$N = 0.025$ N or, $\frac{N}{40}$

81.(a) $Fe_2O_3 + 3CO \rightarrow 2Fe + 3CO_2$

1 mole 3 moles

1 mole of Fe_2O_3 requires 3 mole of CO

Since 1g mole of any gas at NTP occupies 22.42 at NTP 3 mole CO = 3 × 22.4 = 67.2 or 67.2 dm^3

82.(a) Put $y = xe^x$

$\therefore$ $dy = (e^x + xe^x)\,dx = e^x(1 + x)\,dx$

Then $\int \frac{(x + 1)\,e^x dx}{\cos^2(xe^x)}$

$= \int \frac{dy}{\cos^2 y} = \int \sec^2 y\,dy = \tan y + c$

$= \tan(xe^x) + c$

83.(c) Given $y = \sin x$

$\frac{dy}{dx} = \cos x = \sin\left(\frac{\pi}{2} + x\right)$

$\frac{d^2y}{dx^2} = -\sin x = \sin(\pi + x) = \sin\left(2.\frac{\pi}{2} + x\right)$

$\frac{d^3y}{dx^3} = -\cos x = \sin\left(3\,\frac{\pi}{2} + x\right)$

and so on $\frac{d^ny}{dx^n} = \sin\left(n\frac{\pi}{2} + x\right)$

84.(c) $2\sin^{-1}\frac{1}{2} + \cos^{-1}\frac{1}{2}$

$= \sin^{-1}\frac{1}{2} + \sin^{-1}\frac{1}{2} + \cos^{-1}\frac{1}{2}$

$= \frac{\pi}{6} + \frac{\pi}{2} = \frac{\pi + 3\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3}$

85.(a) $\frac{\sin A}{\sin C} = \frac{\sin(A - B)}{\sin(B - C)}$

$\Rightarrow \frac{\sin(B + C)}{\sin(A + B)} = \frac{\sin(A - B)}{\sin(B - C)}$

$\Rightarrow \sin^2 B - \sin^2 C = \sin^2 A - \sin^2 B$

$\Rightarrow b^2 - c^2 = a^2 - b^2 \quad \Rightarrow b^2 - a^2 = c^2 - b^2$

86.(a) Give $\left|\frac{z - 5i}{z + 5i}\right| = 1$

or, $|z - 5i| = |z + 5i|$

or, $|x + iy - 5i| = |x + iy + 5i|$

or, $\sqrt{x^2 + (y - 5)^2} = \sqrt{x^2 + (y + 5)^2}$

or, $x^2 + y^2 - 10y + 25 = x^2 + y^2 + 10y + 25$

or, $y = 0$

$\Rightarrow$ Real axis

87.(a) Let α and β be the roots

Then $A = \frac{\alpha + \beta}{2} \Rightarrow \alpha + B = 2A$

$G = \sqrt{\alpha\beta} \Rightarrow \alpha\beta = G^2$

$\therefore$ Quadratic equation is $x^2 - 2A + G^2 = 0$

88.(b) $S_n = \frac{1}{2}\Sigma t_n = \frac{1}{2}(\Sigma n^2 + \Sigma n)$

$= \frac{1}{2}\left[\frac{n(n + 1)(2n + 1)}{6} + \frac{n(n + 1)}{2}\right]$

$= \frac{1}{2}\frac{n(n + 1)}{2} \cdot \left[\frac{2n + 1}{3} + 1\right] = \frac{n(n + 1)(n + 2)}{6}$

89.(b) $a^2 + b^2 = 7ab$

$\Rightarrow a^2 + 2ab + b^2 = 9ab$

$\Rightarrow \left(\frac{a + b}{3}\right)^2 = ab$

$\Rightarrow \log\left(\frac{a + b}{3}\right)^2 = \log ab$

$\Rightarrow 2\log\left(\frac{a + b}{3}\right) = \log ab$

$\Rightarrow \log\left(\frac{a + b}{3}\right) = \frac{1}{2}\log ab$

90.(c) As the vectors are coplanar, we have $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0 \Rightarrow abc + 2 = a + b + c$

91.(a) No of arrangement of letters of word CALCUITA $= \frac{8!}{2!\,2!\,2!}$

No of arrangements of letters of word AMERICA $= \frac{7!}{2!}$

92.(d) Making homogenous $x^2 + y^2 - 2y + \lambda = 0$

With the help of $x + y = 1$,

So $x^2 + y^2 - 2y(x + y) + \lambda(x + y)^2 = 0$

i.e. $(1 + \lambda)x^2 + y^2(-1 + \lambda) - 2(-\lambda)xy = 0$

The lines are perpendicular if $1 + \lambda - 1 + \lambda = 0$

$\Rightarrow \lambda = 0$

93.(c) The equation of plane at a constant distance of 3p units from the origin is $lx + my + nz = 3p$

Then the coordinates of A, B, C are $\left(\frac{3p}{l}, 0, 0\right)$, $\left(0, \frac{3p}{m}, 0\right)$ and $\left(0, 0, \frac{3p}{n}\right)$. Let $(\bar{x}, \bar{y}, \bar{z})$ be the centroid of ΔABC. Then $\bar{x} = \frac{\frac{3p}{l} + 0 + 0}{3}$

$\Rightarrow \frac{1}{x} = \frac{l}{p}, \frac{1}{y} = \frac{n}{p}, \frac{1}{z} = \frac{n}{p}.$

So, $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{l^2 + m^2 + n^2}{p^2} = \frac{1}{p^2}$

So locus is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}$

94.(a) e is the eccentricity of hyperbola, so $e_1^2 = 1 + \frac{b^2}{a^2}$

$\Rightarrow \frac{1}{e^2} = \frac{a^2}{a^2 + b^2}$

e_1 is the eccentricity of conjugate.

So $e_1^2 = 1 + \frac{a^2}{b^2} \Rightarrow \frac{1}{e_1^2} = \frac{b^2}{a^2 + b^2}$

Then $\frac{1}{e^2} + \frac{1}{e_1^2} = \frac{a^2}{a^2 + b^2} + \frac{b^2}{a^2 + b^2} = 1$

95.(b) Here $\frac{dx}{dt} = 1.2$ m/s $\quad \frac{dy}{dt} = ?$

From similar Δs, $\frac{1.8}{4.5} = \frac{y}{x + y}$

$\Rightarrow 2x + 2y = 5y$

$\Rightarrow 3y = 2x \Rightarrow 3\frac{dy}{dt} = 2\frac{dx}{dt} \Rightarrow \frac{dy}{dt} = 0.8$

96.(d) Required area $= 2\int_0^{\pi} \sin x\, dx$ (By symmetry)

$= 2[-\cos x]_0^{\pi}$

$= 2.[-\cos\pi + \cos 0]$

$= 2[1 + 1] = 4$ sq. units

97.(b) 98.(c) 99.(c) 100.(c)

…Best of Luck…