PEA's

TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

B.E. Model Entrance Exam

2079

Date: 2079-11-20

Hints and Solutions

Section – I

1.(a) Eq^n when pendulum starts from entreme position, is

$y = A\cos\omega t$

or, $\frac{A}{2} = A\cos\left(\frac{2\pi}{T} \times t\right)$

or, $\cos\left(\frac{2\pi}{T} \times t\right) = \frac{1}{2} = \cos 60° = \cos\frac{\pi}{3}$

or, $\frac{2\pi}{T} t = \frac{\pi}{3}$ $\quad t = \frac{T}{6}$

2.(b)

3.(b) % increase $= \frac{\Delta l}{l} \times 100\%$

$= \alpha\Delta\theta \times 100\%$

$= 10^{-5} \times 100 \times 100\% = 0.1\%$

4.(a) Sudden compression $\Rightarrow$ adiabatic process

$P_2V_2^{\gamma} = P_1V_1^{\gamma}$

or, $P_2\left(\frac{m}{d_2}\right)^{\gamma} = P_1\left(\frac{m}{d_1}\right)^{\gamma}$

or, $P_2 = P_1\left(\frac{nd_1}{d_1}\right)^{\gamma}$ $(\because d_2 = nd_1)$

or, $\boxed{P_2 = n^{\gamma}P_1}$

5.(c) $S = \frac{dQ}{m.d\theta}$

While boiling, temperature doesn't change, i.e. $d\theta = 0$, $\Rightarrow S = \infty$

6.(d) Sound wave is longitudinal wave which cant be polarized.

7.(d) $\lambda_m = \frac{\lambda_v}{\mu}$

Since, $\mu > 1$, $\lambda_m < \lambda_v$, i.e. wave length decreases but frequency is not affected.

8.(c) Two particles will have same velocity after a complete wave.

9.(a) $\phi = \frac{Q}{\varepsilon_0}$

Flux is independent to size but depends on charge so,

$\phi' = \frac{\phi}{2\varepsilon_0} = \frac{\phi}{2}$

10.(d) When resistance is placed parallel with voltmeter then resistance decreases current increases so ammeter reading increases & voltmeter reading decreases.

11.(c) Breaking stress $= \frac{F}{A}$

or, F = Breaking stress × A

or, $F \propto A$

The load which can be supported by cable depends on area of cross-section remains constant.

12.(c)

13.(b) Semiconductor have –ve coefficient of resistance, so as temperature increases resistance decreases.

14.(b) $nf = KE + \phi$

or, $eV_s = hf - \phi$

or, $V_s = \frac{hf}{e} - \frac{\phi}{e}$

Which is in form, $y = mx + c$

Where, $m = \frac{h}{e}$

15.(b) Smaller the critical angle, more sparkling. Dipping diamond in water increases critical angle since refractive index decreases.

16.(b) $E_C - E_A = E_C - E_B + E_B - E_A$

or, $\frac{hc}{\lambda_3} = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2}$

$\Rightarrow \frac{1}{\lambda_3} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$

17.(b) $F = \frac{G(M-m)m}{R^2}$

F will be maximum if $\frac{dF}{dm} = 0$

$M - 2m = 0$

$m = \frac{M}{2}$

18.(a) As halogens are most electronegative so configuration is ns^2np^5.

19.(d) CO = 6+8 =14, O_2^{++} = 16-2=14, N_2=2×7=14, Si = 14

20.(b) For M shell, n=3. Hence, no. of orbitals = $n^2 = 3^2 = 9$

21.(c) In $[Fe(H_2O)_5NO]$, NO^+ = 1, H_2O = 0, so Fe has +1.

22.(c) Mass of water = 1gm

Mole of water = $\frac{1}{18}$

Molecules = $\frac{1}{18} \times N_A = \frac{1}{18} \times 6.023 \times 10^{23} = 3.34 \times 10^{22}$

23.(a) $H_2O + SO_2 \rightarrow H_2SO_3$ (Sulphurous acid)

24.(b) ethyne

25.(c) 2-butyne (CH_3-C ≡ C-CH_3) doesn't contain acidic H-atom so it doesn't give ppt with Tollen's reagent.

26.(a) In absence of peroxide electrophillic addition is observed. The first step is addition of H^+ to alkene.

27.(a)

28.(b)

29.(b) Since, $f(-x) = \frac{\sin^4(-x) + \cos^4(-x)}{-x + \tan(-x)}$

$= \frac{\sin^4(x) + \cos^4(x)}{-(x + \tan x)} = -f(x)$

$\Rightarrow$ f(x) is odd function.

30.(a) $n(A\cup B) = n(A) + n(B) - n(A\cap B)$

$n(A\cup B)$ is maximum when $n(A\cap B) = 0$

So, $n(A\cup B) = n(A) + n(B) = 145 > n(U)$, which is not possible.

So, $[n(A\cup B)]_{max} = 125$

31.(a)

32.(b) $\sin^2A + \sin^2B = \sin^2C$

$\Rightarrow \frac{a^2}{k^2} + \frac{b^2}{k^2} = \frac{c^2}{k^2}$

$\Rightarrow a^2 + b^2 = c^2$

$\Rightarrow \Delta$ is rt. angled at C

33.(a) Since, product of root $= \frac{c}{a} = \frac{1}{1}$

$\Rightarrow \alpha.\beta = 1 \quad \Rightarrow \beta = \frac{1}{\alpha}$

34.(d) Projection of $\vec{a}$ on $\vec{b} = a\cos\theta = |\vec{a}|\cos\theta$

Now, $\vec{a}.\vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta$

or, $|\vec{a}|\cos\theta = \dfrac{\vec{a}.\vec{b}}{|\vec{b}|}$

35.(b) Area of Δ made by line ax + by + c = 0 with coordinate axis is $\dfrac{c^2}{2|ab|}$

$= \dfrac{1.p^2}{2(\sin\alpha\cos\alpha)}$

$= \dfrac{p^2}{|\sin^2\alpha|}$

36.(b) $x^2 + 5x + 6 = 0$

$\Rightarrow$ $x = -2$

$x = -3$

Pair of planes parallel to yz plane.

37.(d) Length of L.R. = 4 × distance between vertex and focus

$= 4 \times 2a = 8a$

38.(c) $\lim_{x \to \infty} \dfrac{x^{20}}{e^x}$

Using L' Hospital rule upto 20th time

$\lim_{x \to \infty} \dfrac{20!}{e^x} = \dfrac{20!}{\infty} = 0$

39.(d) $\int_{-1}^{1} x|x|\,dx = 0$ ($\because$ x|x| = odd function)

40.(c) $\sin^{-1}x + c$

(Derivative and anti-derivative are inverse of each other so they cancel each other)

41.(c) $y = |x|$

or, $\dfrac{dy}{dx} = \dfrac{x}{|x|}$, at x = 0 is undefined.

42.(b) $y = \sqrt{4 - x^2}$ represent upper half-part of circle $x^2+y^2=4$

So area $= \dfrac{\pi.2^2}{2} = 2\pi$

43.(c) $\cos^{-1}x + \cos^{-1}y \left(\dfrac{\pi}{2} - \sin^{-1}x\right) + \left(\dfrac{\pi}{2} - \sin^{-1}y\right)$

$= \pi - (\sin^{-1}x + \sin^{-1}y)$

$= \pi - \dfrac{\pi}{3} = \dfrac{2\pi}{3}$

44.(a) $\sin^2x + \mathrm{cosec}^2x = 2$

or, $\sin^2x + \dfrac{1}{\sin^2x} = 2$

or, $\sin^4x - 2\sin^2x + 1 = 0d$

or, $(\sin^2x)^2 - 2.\sin^2x.1 + 1^2 = 0$

or, $(\sin^2x - 1) = 0$

or, $\sin^2x = 1$

$\Rightarrow$ $x = n\pi \pm \dfrac{\pi}{2}$

45.(d) $3x + 4y = 12$

$\Rightarrow$ $\dfrac{x}{4} + \dfrac{4}{3} = 1$

So, portion intercepted $= \sqrt{3^2 + 4^2} = 5$

46.(b) Sum of all coefficient $= 2^{10}$

Sum of coefficient of even power of x $= \dfrac{2^{10}}{2} = 2^9$

47.(d) $\dfrac{x}{a} = t + \dfrac{1}{t}, \dfrac{y}{b} = t - \dfrac{1}{t}$

or, $\left(\dfrac{x}{a}\right)^2 - \left(\dfrac{y}{b}\right)^2 = \left(t + \dfrac{1}{t}\right)^2 - \left(t - \dfrac{1}{t}\right)^2$

or, $\left(\dfrac{x}{a}\right)^2 - \left(\dfrac{y}{b}\right)^2 = 4$, i.e. a hyperbola.

48.(b) $\log_e e + \dfrac{\log_e 3}{1!} + \dfrac{(\log_e 3)^2}{2!} + \dfrac{(\log_e 3)^2}{3!} + \ldots \infty$

$= e^{\log_e 3} = 3$

49.(d)	50.(c)	51.(a)	52.(b)	53.(a)	54.(c)
55.(a)	56.(a)	57.(d)	58.(d)	59.(c)	60.(a)

Section – II

61.(c) If a particle covers equal distance in 5th and 6th second, then during 5th second it moves up & during 6th second it moves down so, time to reach man height = 5s

O = u – gt

u = 10 × 5 = 50 m/s

62.(c) Loss in KE = Gain in PE

$\dfrac{1}{2}mv^2 + \dfrac{1}{2}I\omega^2 = mgh$

or, $\dfrac{1}{2}mv^2 + \dfrac{1}{2}mk^2.\dfrac{v^2}{r^2} = mgh$

or, $\dfrac{1}{2}mv^2\left(1 + \dfrac{k^2}{r^2}\right) = mgh$

or, $h = \dfrac{v^2\left(\dfrac{k^2}{r^2} + 1\right)}{2g}$

$= \dfrac{10^2\left(\dfrac{2r^2}{5r^2} + 1\right)}{2 \times 10} = 7$ m

63.(b) $\omega = \omega_s - \omega_e$

$\Rightarrow$ $\dfrac{2\pi}{T} = \dfrac{2\pi}{T_s} - \dfrac{2\pi}{T_e}$

$\Rightarrow$ $\dfrac{1}{T} = \dfrac{1}{3} - \dfrac{1}{24}$

or, $T = \dfrac{24}{7}$ hrs

64.(a) PV = RT for 1 mole

$W = \int PdV \int \dfrac{RT}{V}dV$

Given, $V = CT^{2/3}$

$\Rightarrow$ $dV = C.\dfrac{2}{3}T^{-1/3}\,dT$

$\dfrac{dV}{V} \Rightarrow \dfrac{2}{3}T^{-1}dT$

or, $\dfrac{dV}{V} = \dfrac{2}{3}\dfrac{dT}{T}$

$W = \int_{T_1}^{T_2} RT.\dfrac{2}{3}\dfrac{dT}{T}$

$= \dfrac{2}{3}R(T_2 - T_1)$

$= \dfrac{2R}{3} \times 30 = 20R$

$= 20 \times 8.31 = 166$

65.(b) Radiating power of black body,

$E = 6(T^4 - T_0^4)\,A$

$T_0 = 227°C = 500$ K,

$T_1 = 727°C = 1000$K

$T_2 = 1227°C = 1500$ K

$E_1 = \sigma(1000^4 - 500^4)$ … (i)

$E_2 = \sigma(1500^4 - 500^4)$ … (ii)

Dividing, $\frac{60}{E_2} = \frac{1000^4 - 500^4}{1500^4 - 500^4}$, $\boxed{E_2 = 320\ W}$

66.(c) $\gamma = \alpha_1 + \alpha_2 + \alpha_3$
$= \alpha_1 + 2\alpha_2$

67.(a) The delector receives direct as well as reflected waves. Distance moved between two consecutive position of maxima $\frac{\lambda}{2}$

For 14 successive maxima = $14 \times \frac{\lambda}{2}$

Given, $14 \times \frac{\lambda}{2} = 0.14$

or, $\lambda = 2 \times 10^{-2}$ m

$\therefore\ f = \frac{c}{\lambda} = \frac{3 \times 10^8}{2 \times 10^{-2}} = 1.5 \times 10^{10}$ Hz

68.(b) The slope of image is m = tan135° = –1
Equation of line through origin, y = mx
$y = -x$
$y + x = 0$

69.(a) Maxima is at P

$\frac{xd}{D} = n\lambda$

or, $\frac{d}{2} \cdot \frac{d}{D} = n\lambda$

or, $n = \frac{d^2}{2\lambda D}$

70.(d) $\vec{E}_x = -\frac{\partial V}{\partial_x}\hat{i} = -(2xy - z^3)\hat{i} = (2xy + z^3)\hat{i}$

$\vec{E}_y = -\frac{\partial V}{\partial_y}\hat{j} = -(-x^2)\hat{j} = x^2\hat{j}$

$\vec{E}_z = -\frac{\partial V}{\partial_z}\hat{k} = -(-3xz^2)\hat{k} = 3xz^2\hat{k}$

$\therefore\ \vec{E} = \vec{E}_x + \vec{E}_y + \vec{E}_z = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$

71.(c) $B_R = B_2 - B_1 = \frac{\mu_0 I_2}{2\pi.\frac{r}{2}} - \frac{\mu_0 I_1}{2\pi.\frac{r}{2}}$

$= \frac{\mu_0}{\pi \times 5}(5 - 2.5) = \frac{\mu_0}{2\pi}$

72.(d) $E = -\frac{d\phi}{dt}$

$E = -\frac{d\phi}{dt}$

or, $IR = -\frac{d}{dt}(6t^2 - 5t + 1)$

or, $I = -\frac{(12t - 5)}{R}$

When t = 0.25s

Then $I = \frac{2}{10} = 0.2A$

73.(d) $X^3 + Y^5 \rightarrow 2Z^4$
AE = [3 × 5.3 + 5 × 7.4) – 2(4 × 6.2) = 3.3 MeV
Hence correct energy is option (d)

74.(b) $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$, n = no of decays

$\frac{1}{256} = \left(\frac{1}{2}\right)^n$

$\Rightarrow$ n = 8 haf lifes
Times for 8 half lives = 8 × 12.5 = 100 hrs

75.(a) $M = \frac{E}{F} \times It$

or, $500 = \frac{9}{96500} \times 25 \times t$

t = 214444.4 sec = 59.56 hrs

76.(c) $N_{mixture} = \frac{300 \times 10^{-2} - 200 \times 10^{-3}}{300 + 200}$

$= 5.6 \times 10^{-3}$ N (w.r.t base)
pOH = -log $(5.6 \times 10^{-3}) = 2.25$
pH = 14-2.25 = 11.75

77.(c) $C_2H_5Cl + Mg \xrightarrow{Dry\ Ether} C_2H_5MgCl \xrightarrow{H2O} C_2H_6 + MgCl.OH$

78.(a) $K_C = \frac{[P_{CO}]^2}{[P_{CO_2}]} = \frac{8^2}{4} = 16$ atm

79.(c) % of Haemoglobin = 0.33
wt of Iron = $67200 \times \frac{0.33}{100} = 221.76$
So, No. of Fe atoms = $\frac{221.76}{56} = 3.96 \sim 4$

80.(d) No. of mole of $CO_2 = \frac{88}{44} = 2$ mole
2 mole CO_2 contain 4 mole oxygen atom.
1 mole CO contain 1 mole oxygen atom.
So, 4 mole CO contain 4 mole oxygen atom.
4 mole CO = 4 × (12 +16) = 112 gm

81.(a) $NaHSO_3 + NaHS \rightarrow Na_2S_2O_3 + H_2O$
$Na_2S_2O_3 + HCl \rightarrow NaCl + H_2O + SO_2 + S\downarrow$
colloidal

82.(d) $\left|\frac{(3 + 4i)(\sin\theta + i\cos\theta)}{\sin\theta - i\cos\theta}\right|$

$= \frac{|3 + 4i|\ |\sin\theta + i\cos\theta|}{|\sin\theta - i\cos\theta|}$

$= \frac{(3^2 + 5^2).(1)}{(1)} = 5$

83.(c) For f(x) to be defined,
$|x| - x > 0$
or, $x < |x|$, which is true for all $x \in (-\infty, 0)$

84.(b) Given equation can be written as
$5^{3x} + 45^x = 2.3^{3x}$

or, $\left(\frac{5}{3}\right)^{3x} + \left(\frac{5}{3}\right)^x = 2$

Let, $\left(\frac{5}{3}\right)^x = t$

$\Rightarrow$ $t^3 + t - 2 = 0$
or, $t^3 - 1 + t - 1 = 0$
or, $(t - 1)(t^2 + t + 1) + (t - 1) = 0$
or, $(t - 1)(t^2 + t + 2) = 0$
$\Rightarrow$ t = 1
or, $t^2 + t + 2 = 0$
But, $t^2 + t + 2 = 0$ does not have real solutions
$\therefore$ t = 1

$\Rightarrow$ $\left(\frac{5}{3}\right)^x = 1$ $\Rightarrow$ x = 0, one solution only

85.(b) $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$

or, $\frac{\cos A}{2R\sin A} = \frac{\cos B}{2R\sin B} = \frac{\cos C}{2R\sin C}$

or, cotA = cotB = cotC
$\Rightarrow$ A = B = C
$\Rightarrow$ Δ is equilateral

$\therefore$ Area = $\frac{\sqrt{3}}{4} l^2 = \frac{\sqrt{3}}{4} \times \frac{1}{6} = \frac{1}{8\sqrt{3}}$

86.(a) Given, $\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$

or, $\tan^{-1}\left(\frac{2x + 3x}{1 - 2x.3x}\right) = \tan^{-1}(1)$

or, $\frac{5x}{1 - 6x^2} = 1$

or, $6x^2 + 5x - 1 = 0 \qquad x = \frac{1}{6}, -1$

But x = –1 is in option

87.(c) Given, $(1 + x - 2x^2)^6 = 1 + a_1x + a_2x^2 + \ldots + a_{12}x^{12}$

Putting x = 1, we get

$0 = 1 + a_1 + a_2 + \ldots + a_{12} \ldots(1)$

Putting x = –1, we get

$64 = 1 - a_1 + a_2 - \ldots + a_{12} \ldots (2)$

Adding (1) & (2)

$64 = 2(1 + a_2 + a_4 + \ldots)$

or, $a_2 + a_4 + \ldots a_{12} = 31$

88.(c) Let $\tan^{-1}x = y$

$\Rightarrow \frac{1}{1 + x^2}dx = dy$ and $x = \tan y$

$= \int e^y.(1 + \tan y + \tan^2 y).dy$

$= \int e^y(\tan y + \sec^2 y)\,dy$

$= e^y \tan y + c$

$= t^{\tan^{-1}} x.\tan(\tan^{-1}x) + c = e^{\tan^{-1}x}\, x + c$

89.(c) Since, $(1 - x)^{-1} = 1 + x + x^2 + \ldots$

$\Rightarrow x = \frac{1}{1 - a}, y = \frac{1}{1 - b}, z = \frac{1}{1 - c}$

Since, a, b, c are in A.P.

$\Rightarrow$ 1 – a, 1 – b, 1 – c are in A.P.

$\Rightarrow \frac{1}{1 - a}, \frac{1}{1 - b}, \frac{1}{1 - c}$ are in H.P.

$\Rightarrow$ x, y, z are in H.P.

90.(b) $y = \tan^{-1}\left(\frac{5x - x}{1 + 5x.x}\right) + \tan^{-1}\left(\frac{x + \frac{2}{3}}{1 - \frac{2}{3}x}\right)$

or, $y = \tan^{-1}(5x) - \tan^{-1}(x) + \tan^{-1}(x) + \tan^{-1}\left(\frac{2}{3}\right)$

or, $y = \tan^{-1}(5x) + \tan^{-1}\left(\frac{2}{3}\right)$

or, $\frac{dy}{dx} = \frac{5}{1 + 25x^2}$

91.(d) Area $= \int_0^{\pi/4} \tan x + \int_{\pi/4}^{\pi/2} \cot x\,dx$

$= \log(\sec x)\Big|_0^{\pi/4} + \log(\sin x)\Big|_{\pi/4}^{\pi/2}$

$= \log\left(\frac{\sec\frac{\pi}{4}}{\sec 0}\right) + \log\left(\frac{\sin\frac{\pi}{2}}{\sin\frac{\pi}{4}}\right) = \log(\sqrt{2}) + \log\left(\frac{1}{\frac{1}{\sqrt{2}}}\right)$

$= \log\sqrt{2} + \log\sqrt{2}$

$= 2\log\sqrt{2}$

$= \log 2$

So, answer are both a and b

92.(a) $(\vec{a} + \vec{b}).\vec{b} = 0$

or, $\vec{a}.\vec{b} + \vec{b}.\vec{b} = 0$

or, $\vec{a}.\vec{b} = -\vec{b}.\vec{b} \ldots (1)$

$(\vec{a} + 2\vec{b}).\vec{a} = 0$

or, $\vec{a}.\vec{a} + 2\vec{b}.\vec{a} = 0$

or, $\vec{a}.\vec{a} + 2\vec{a}.\vec{b} = 0$

or, $\vec{a}.\vec{a} - 2\vec{b}.\vec{b} = 0$

or, $|\vec{a}|^2 = 2.|\vec{b}|^2$

or, $|\vec{a}| = \sqrt{2}.|\vec{b}|$

93.(c) $y = (x^2 - 1)(x^2 - 5) = x^4 - 6x^2 + 5$

$\frac{dy}{dx} = 4x^3 - 12x$

$\frac{d^2y}{dx^2} = 12x^2 - 12$

For curve to be concave upwards

$f''(x) > 0$

i.e. $12(x - 1)(x + 1) > 0$

or, $(x - 1)(x + 1) > 0$

i.e. $x < -1$ or $x > 1$

$\Rightarrow |x| > 1$

94.(a) Given circle, $(x - 6)^2 + y^2 = 2$

Equation of tangent is, $Y = mX + a\sqrt{1 + m^2}$

Where Y = y, X = x – 6 for this question

or, $y = m(x - 6) + \sqrt{2}\sqrt{1 + m^2}$

or, $y = m(x - 6) + \sqrt{2(1 + m^2)}$

Focal point of $y^2 = 16x$ is (a, 0) = (4, 0)

Now, focal chord is tangent to circle, so focal point must satisfy equation of tangent so

$0 = m(4 - 6) + \sqrt{2(1 + m^2)}$

or, $2m = \sqrt{2(1 + m^2)}$

or, $4m^2 = 2 + 2m^2$

or, $m^2 = 1$

or, $m = \pm 1$

95.(c) On, y-axis, x = 0 equation of circle becomes

$y^2 + y - 20 = 0$

$\Rightarrow y = -5$ & 4

So, circle touch y axis at (0, –5) & (0, 4)

Hence, intercept length $\Rightarrow |-5 - 4| = 9$

96.(c) Centroid, $x = \frac{a\cos t + b\sin t + 1}{3}$

$\Rightarrow a\cos t + b\sin t = 3x - 1 \ldots(1)$

$y = \frac{a\sin t - b\cos t}{3}$

$\Rightarrow a\sin t - b\cos t = 3y \ldots (2)$

Squaring & adding (1) & (2)

$a^2\cos^2 t + 2ab\cos t\sin t + b^2\sin^2 t + a^2\sin^2 t - 2ab\cos t\sin t + b^2\cos^2 t$

$= (3x - 1)^2 + (3y)^2$

or, $a^2(\cos^2 + \sin^2 t) + b^2(\sin^2 t + \cos^2 t) = (3x - 1)^2 + (3y)^2$

or, $(3x - 1)^2 + (3y)^2 = a^2 + b^2$

97.(c) 98.(a) 99.(d) 100.(a)

…Best of Luck…

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or, $\cos\left(\frac{2\pi}{T} \times t\right) = \frac{1}{2} = \cos 60° = \cos\frac{\pi}{3}$

or, $\frac{2\pi}{T} t = \frac{\pi}{3}$ $\quad t = \frac{T}{6}$

2.(b)

3.(b) % increase $= \frac{\Delta l}{l} \times 100\%$

$= \alpha\Delta\theta \times 100\%$

$= 10^{-5} \times 100 \times 100\% = 0.1\%$

4.(a) Sudden compression $\Rightarrow$ adiabatic process

$P_2V_2^\gamma = P_1V_1^\gamma$

or, $P_2\left(\frac{m}{d_2}\right)^\gamma = P_1\left(\frac{m}{d_1}\right)^\gamma$

or, $P_2 = P_1\left(\frac{nd_1}{d_1}\right)^\gamma$ $(\because d_2 = nd_1)$

or, $\boxed{P_2 = n^\gamma P_1}$

5.(c) $S = \frac{dQ}{m.d\theta}$

While boiling, temperature doesn't change, i.e. $d\theta = 0$, $\Rightarrow S = \infty$

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7.(d) $\lambda_m = \frac{\lambda_v}{\mu}$

Since, $\mu > 1$, $\lambda_m < \lambda_v$, i.e. wave length decreases but frequency is not affected.

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or, $\frac{hc}{\lambda_3} = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2}$

$\Rightarrow$ $\frac{1}{\lambda_3} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$

17.(b) $F = \frac{G(M - m)m}{R^2}$

F will be maximum if $\frac{dF}{dm} = 0$

$M - 2m = 0$

$m = \frac{M}{2}$

18.(a) As halogens are most electronegative so configuration is ns^2np^5.

19.(d) CO = 6+8 =14, O_2^{++} = 16-2=14, N_2=2×7=14, Si = 14

20.(b) For M shell, n=3. Hence, no. of orbitals = $n^2 = 3^2 = 9$

21.(c) In $[Fe(H_2O)_5NO]$, NO^+ = 1, H_2O = 0, so Fe has +1.

22.(c) Mass of water = 1gm

Mole of water = $\frac{1}{18}$

Molecules = $\frac{1}{18} \times N_A = \frac{1}{18} \times 6.023 \times 10^{23} = 3.34 \times 10^{22}$

23.(a) $H_2O + SO_2 \rightarrow H_2SO_3$ (Sulphurous acid)

24.(b) ethyne

25.(c) 2-butyne (CH_3-C ≡ C-CH_3) doesn't contain acidic H-atom so it doesn't give ppt with Tollen's reagent.

26.(a) In absence of peroxide electrophillic addition is observed. The first step is addition of H^+ to alkene.

27.(a)

28.(b)

29.(b) Since, $f(-x) = \frac{\sin^4(-x) + \cos^4(-x)}{-x + \tan(-x)}$

$= \frac{\sin^4(x) + \cos^4(x)}{-(x + \tan x)} = -f(x)$

$\Rightarrow$ f(x) is odd function.

30.(a) $n(A\cup B) = n(A) + n(B) - n(A\cap B)$

$n(A\cup B)$ is maximum when $n(A\cap B) = 0$

So, $n(A\cup B) = n(A) + n(B) = 145 > n(U)$, which is not possible.

So, $[n(A\cup B)]_{max} = 125$

31.(a)

32.(b) $\sin^2A + \sin^2B = \sin^2C$

$\Rightarrow$ $\frac{a^2}{k^2} + \frac{b^2}{k^2} = \frac{c^2}{k^2}$

$\Rightarrow$ $a^2 + b^2 = c^2$

$\Rightarrow$ Δ is rt. angled at C

33.(a) Since, product of root $= \frac{c}{a} = \frac{1}{1}$

$\Rightarrow$ $\alpha.\beta = 1$ $\quad\Rightarrow \beta = \frac{1}{\alpha}$

34.(d) Projection of $\vec{a}$ on $\vec{b} = a\cos\theta = |\vec{a}|\cos\theta$

Now, $\vec{a}.\vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta$

or, $|\vec{a}|\cos\theta = \dfrac{\vec{a}.\vec{b}}{|\vec{b}|}$

35.(b) Area of Δ made by line ax + by + c = 0 with coordinate axis is $\dfrac{c^2}{2|ab|}$

$= \dfrac{1.p^2}{2(\sin\alpha\cos\alpha)}$

$= \dfrac{p^2}{|\sin^2\alpha|}$

36.(b) $x^2 + 5x + 6 = 0$

$\Rightarrow$ $x = -2$

$x = -3$

Pair of planes parallel to yz plane.

37.(d) Length of L.R. = 4 × distance between vertex and focus

$= 4 \times 2a = 8a$

38.(c) $\lim\limits_{x \to \infty} \dfrac{x^{20}}{e^x}$

Using L' Hospital rule upto 20th time

$\lim\limits_{x \to \infty} \dfrac{20!}{e^x} = \dfrac{20!}{\infty} = 0$

39.(d) $\int_{-1}^{1} x|x|\,dx = 0$ ($\because$ x|x| = odd function)

40.(c) $\sin^{-1}x + c$

(Derivative and anti-derivative are inverse of each other so they cancel each other)

41.(c) $y = |x|$

or, $\dfrac{dy}{dx} = \dfrac{x}{|x|}$, at x = 0 is undefined.

42.(b) $y = \sqrt{4 - x^2}$ represent upper half-part of circle $x^2+y^2=4$

So area $= \dfrac{\pi.2^2}{2} = 2\pi$

43.(c) $\cos^{-1}x + \cos^{-1}y\left(\dfrac{\pi}{2} - \sin^{-1}x\right) + \left(\dfrac{\pi}{2} - \sin^{-1}y\right)$

$= \pi - (\sin^{-1}x + \sin^{-1}y)$

$= \pi - \dfrac{\pi}{3} = \dfrac{2\pi}{3}$

44.(a) $\sin^2x + \text{cosec}^2x = 2$

or, $\sin^2x + \dfrac{1}{\sin^2x} = 2$

or, $\sin^4x - 2\sin^2x + 1 = 0d$

or, $(\sin^2x)^2 - 2.\sin^2x.1 + 1^2 = 0$

or, $(\sin^2x - 1) = 0$

or, $\sin^2x = 1$

$\Rightarrow$ $x = n\pi \pm \dfrac{\pi}{2}$

45.(d) $3x + 4y = 12$

$\Rightarrow$ $\dfrac{x}{4} + \dfrac{4}{3} = 1$

So, portion intercepted $= \sqrt{3^2 + 4^2} = 5$

46.(b) Sum of all coefficient $= 2^{10}$

Sum of coefficient of even power of x $= \dfrac{2^{10}}{2} = 2^9$

47.(d) $\dfrac{x}{a} = t + \dfrac{1}{t}, \dfrac{y}{b} = t - \dfrac{1}{t}$

or, $\left(\dfrac{x}{a}\right)^2 - \left(\dfrac{y}{b}\right)^2 = \left(t + \dfrac{1}{t}\right)^2 - \left(t - \dfrac{1}{t}\right)^2$

or, $\left(\dfrac{x}{a}\right)^2 - \left(\dfrac{y}{b}\right)^2 = 4$, i.e. a hyperbola.

48.(b) $\log_e e + \dfrac{\log_e 3}{1!} + \dfrac{(\log_e 3)^2}{2!} + \dfrac{(\log_e 3)^2}{3!} + \ldots\ \infty$

$= e^{\log_e 3} = 3$

49.(d)	50.(c)	51.(a)	52.(b)	53.(a)	54.(c)
55.(a)	56.(a)	57.(d)	58.(d)	59.(c)	60.(a)

Section – II

61.(c) If a particle covers equal distance in 5th and 6th second, then during 5th second it moves up & during 6th second it moves down so, time to reach man height = 5s

$O = u - gt$

$u = 10 \times 5 = 50$ m/s

62.(c) Loss in KE = Gain in PE

$\dfrac{1}{2}mv^2 + \dfrac{1}{2}I\omega^2 = mgh$

or, $\dfrac{1}{2}mv^2 + \dfrac{1}{2}mk^2.\dfrac{v^2}{r^2} = mgh$

or, $\dfrac{1}{2}mv^2\left(1 + \dfrac{k^2}{r^2}\right) = mgh$

or, $h = \dfrac{v^2\left(\dfrac{k^2}{r^2} + 1\right)}{2g}$

$= \dfrac{10^2\left(\dfrac{2r^2}{5r^2} + 1\right)}{2 \times 10} = 7$ m

63.(b) $\omega = \omega_s - \omega_e$

$\Rightarrow$ $\dfrac{2\pi}{T} = \dfrac{2\pi}{T_s} - \dfrac{2\pi}{T_e}$

$\Rightarrow$ $\dfrac{1}{T} = \dfrac{1}{3} - \dfrac{1}{24}$

or, $T = \dfrac{24}{7}$ hrs

64.(a) PV = RT for 1 mole

$W = \int PdV \int \dfrac{RT}{V}dV$

Given, $V = CT^{2/3}$

$\Rightarrow$ $dV = C.\dfrac{2}{3}T^{-1/3}\,dT$

$\dfrac{dV}{V} \Rightarrow \dfrac{2}{3}T^{-1}dT$

or, $\dfrac{dV}{V} = \dfrac{2}{3}\dfrac{dT}{T}$

$W = \int_{T_1}^{T_2} RT.\dfrac{2}{3}\dfrac{dT}{T}$

$= \dfrac{2}{3}R(T_2 - T_1)$

$= \dfrac{2R}{3} \times 30 = 20R$

$= 20 \times 8.31 = 166$

65.(b) Radiating power of black body,

$E = 6(T^4 - T_0^4)\,A$

$T_0 = 227°C = 500$ K,

$T_1 = 727°C = 1000$K

$T_2 = 1227°C = 1500$ K

$E_1 = \sigma(1000^4 - 500^4) \ldots$ (i)

$E_2 = \sigma(1500^4 - 500^4)$ … (ii)

Dividing, $\frac{60}{E_2} = \frac{1000^4 - 500^4}{1500^4 - 500^4}$, $\boxed{E_2 = 320\text{ W}}$

66.(c) $\gamma = \alpha_1 + \alpha_2 + \alpha_3$
$= \alpha_1 + 2\alpha_2$

67.(a) The delector receives direct as well as reflected waves. Distance moved between two consecutive position of maxima $\frac{\lambda}{2}$

For 14 successive maxima = $14 \times \frac{\lambda}{2}$

Given, $14 \times \frac{\lambda}{2} = 0.14$

or, $\lambda = 2 \times 10^{-2}$ m

$\therefore\quad f = \frac{c}{\lambda} = \frac{3 \times 10^8}{2 \times 10^{-2}} = 1.5 \times 10^{10}$ Hz

68.(b) The slope of image is m = tan135° = –1
Equation of line through origin, y = mx
y = –x
y + x = 0

69.(a) Maxima is at P

$\frac{xd}{D} = n\lambda$

or, $\frac{d}{2}\frac{d}{\cdot D} = n\lambda$

or, $n = \frac{d^2}{2\lambda D}$

70.(d) $\vec{E}_x = -\frac{\partial V}{\partial_x}\hat{i} = -(2xy - z^3)\hat{i} = (2xy + z^3)\hat{i}$

$\vec{E}_y = -\frac{\partial V}{\partial_y}\hat{j} = -(-x^2)\hat{j} = x^2\hat{j}$

$\vec{E}_z = -\frac{\partial V}{\partial_z}\hat{k} = -(-3xz^2)\hat{k} = 3xz^2\hat{k}$

$\therefore\quad \vec{E} = \vec{E}_x + \vec{E}_y + \vec{E}_z = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$

71.(c) $B_R = B_2 - B_1 = \frac{\mu_0 I_2}{2\pi.\frac{r}{2}} - \frac{\mu_0 I_1}{2\pi.\frac{r}{2}}$

$= \frac{\mu_0}{\pi \times 5}(5 - 2.5) = \frac{\mu_0}{2\pi}$

72.(d) $E = -\frac{d\phi}{dt}$

$E = -\frac{d\phi}{dt}$

or, $IR = -\frac{d}{dt}(6t^2 - 5t + 1)$

or, $I = -\frac{(12t - 5)}{R}$

When t = 0.25s

Then $I = \frac{2}{10} = 0.2$A

73.(d) $X^3 + Y^5 \rightarrow 2Z^4$
AE = [3 × 5.3 + 5 × 7.4) – 2(4 × 6.2) = 3.3 MeV
Hence correct energy is option (d)

74.(b) $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$, n = no of decays

$\frac{1}{256} = \left(\frac{1}{2}\right)^n$

$\Rightarrow$ n = 8 haf lifes
Times for 8 half lives = 8 × 12.5 = 100 hrs

75.(a) $M = \frac{E}{F} \times It$

or, $500 = \frac{9}{96500} \times 25 \times t$

t = 214444.4 sec = 59.56 hrs

76.(c) $N_{mixture} = \frac{300 \times 10^{-2} - 200 \times 10^{-3}}{300 + 200}$

$= 5.6 \times 10^{-3}$ N (w.r.t base)
pOH = -log $(5.6 \times 10^{-3}) = 2.25$
pH = 14-2.25 = 11.75

77.(c) $C_2H_5Cl + Mg \xrightarrow{Dry\ Ether} C_2H_5MgCl \xrightarrow{H2O} C_2H_6 + MgCl.OH$

78.(a) $K_C = \frac{[P_{CO}]^2}{[P_{CO_2}]} = \frac{8^2}{4} = 16$ atm

79.(c) % of Haemoglobin = 0.33
wt of Iron = $67200 \times \frac{0.33}{100} = 221.76$
So, No. of Fe atoms = $\frac{221.76}{56} = 3.96 \sim 4$

80.(d) No. of mole of $CO_2 = \frac{88}{44} = 2$ mole
2 mole CO_2 contain 4 mole oxygen atom.
1 mole CO contain 1 mole oxygen atom.
So, 4 mole CO contain 4 mole oxygen atom.
4 mole CO = 4 × (12 +16) = 112 gm

81.(a) $NaHSO_3 + NaHS \rightarrow Na_2S_2O_3 + H_2O$
$Na_2S_2O_3 + HCl \rightarrow NaCl + H_2O + SO_2 + S\downarrow$
colloidal

82.(d) $\left|\frac{(3 + 4i)(\sin\theta + i\cos\theta)}{\sin\theta - i\cos\theta}\right|$

$= \frac{|3 + 4i|\ |\sin\theta + i\cos\theta|}{|\sin\theta - i\cos\theta|}$

$= \frac{(3^2 + 5^2).(1)}{(1)} = 5$

83.(c) For f(x) to be defined,
$|x| - x > 0$
or, $x < |x|$, which is true for all $x \in (-\infty, 0)$

84.(b) Given equation can be written as
$5^{3x} + 45^x = 2.3^{3x}$

or, $\left(\frac{5}{3}\right)^{3x} + \left(\frac{5}{3}\right)^x = 2$

Let, $\left(\frac{5}{3}\right)^x = t$

$\Rightarrow$ $t^3 + t - 2 = 0$
or, $t^3 - 1 + t - 1 = 0$
or, $(t - 1)(t^2 + t + 1) + (t - 1) = 0$
or, $(t - 1)(t^2 + t + 2) = 0$
$\Rightarrow$ t = 1
or, $t^2 + t + 2 = 0$
But, $t^2 + t + 2 = 0$ does not have real solutions
$\therefore$ t = 1

$\Rightarrow$ $\left(\frac{5}{3}\right)^x = 1$ $\quad\Rightarrow$ x = 0, one solution only

85.(b) $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$

or, $\frac{\cos A}{2R\sin A} = \frac{\cos B}{2R\sin B} = \frac{\cos C}{2R\sin C}$

or, cotA = cotB = cotC
$\Rightarrow$ A = B = C
$\Rightarrow$ Δ is equilateral

$\therefore$ Area $= \frac{\sqrt{3}}{4} l^2 = \frac{\sqrt{3}}{4} \times \frac{1}{6} = \frac{1}{8\sqrt{3}}$

86.(a) Given, $\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$

or, $\tan^{-1}\left(\frac{2x+3x}{1-2x.3x}\right) = \tan^{-1}(1)$

or, $\frac{5x}{1-6x^2} = 1$

or, $6x^2 + 5x - 1 = 0 \qquad x = \frac{1}{6}, -1$

But x = –1 is in option

87.(c) Given, $(1 + x - 2x^2)^6 = 1 + a_1x + a_2x^2 + \ldots + a_{12}x^{12}$

Putting x = 1, we get

$0 = 1 + a_1 + a_2 + \ldots + a_{12} \ldots(1)$

Putting x = –1, we get

$64 = 1 - a_1 + a_2 - \ldots + a_{12} \ldots (2)$

Adding (1) & (2)

$64 = 2(1 + a_2 + a_4 + \ldots)$

or, $a_2 + a_4 + \ldots a_{12} = 31$

88.(c) Let $\tan^{-1}x = y$

$\Rightarrow \frac{1}{1+x^2}dx = dy$ and $x = \tan y$

$= \int e^y.(1 + \tan y + \tan^2 y).dy$

$= \int e^y(\tan y + \sec^2 y)\, dy$

$= e^y \tan y + c$

$= t^{\tan^{-1}}x.\tan(\tan^{-1}x) + c = e^{\tan^{-1}x}\, x + c$

89.(c) Since, $(1 - x)^{-1} = 1 + x + x^2 + \ldots$

$\Rightarrow x = \frac{1}{1-a}, y = \frac{1}{1-b}, z = \frac{1}{1-c}$

Since, a, b, c are in A.P.

$\Rightarrow$ 1 – a, 1 – b, 1 – c are in A.P.

$\Rightarrow \frac{1}{1-a}, \frac{1}{1-b}, \frac{1}{1-c}$ are in H.P.

$\Rightarrow$ x, y, z are in H.P.

90.(b) $y = \tan^{-1}\left(\frac{5x-x}{1+5x.x}\right) + \tan^{-1}\left(\frac{x+\frac{2}{3}}{1-\frac{2}{3}x}\right)$

or, $y = \tan^{-1}(5x) - \tan^{-1}(x) + \tan^{-1}(x) + \tan^{-1}\left(\frac{2}{3}\right)$

or, $y = \tan^{-1}(5x) + \tan^{-1}\left(\frac{2}{3}\right)$

or, $\frac{dy}{dx} = \frac{5}{1+25x^2}$

91.(d) Area $= \int_0^{\pi/4} \tan x + \int_{\pi/4}^{\pi/2} \cot x\, dx$

$= \log(\sec x)\Big|_0^{\pi/4} + \log(\sin x)\Big|_{\pi/4}^{\pi/2}$

$= \log\left(\frac{\sec\frac{\pi}{4}}{\sec 0}\right) + \log\left(\frac{\sin\frac{\pi}{2}}{\sin\frac{\pi}{4}}\right) = \log(\sqrt{2}) + \log\left(\frac{1}{\frac{1}{\sqrt{2}}}\right)$

$= \log\sqrt{2} + \log\sqrt{2}$

$= 2\log\sqrt{2}$

$= \log 2$

So, answer are both a and b

92.(a) $(\vec{a} + \vec{b}).\vec{b} = 0$

or, $\vec{a}.\vec{b} + \vec{b}.\vec{b} = 0$

or, $\vec{a}.\vec{b} = -\vec{b}.\vec{b} \ldots (1)$

$(\vec{a} + 2\vec{b}).\vec{a} = 0$

or, $\vec{a}.\vec{a} + 2\vec{b}.\vec{a} = 0$

or, $\vec{a}.\vec{a} + 2\vec{a}.\vec{b} = 0$

or, $\vec{a}.\vec{a} - 2\vec{b}.\vec{b} = 0$

or, $|\vec{a}|^2 = 2.|\vec{b}|^2$

or, $|\vec{a}| = \sqrt{2}.|\vec{b}|$

93.(c) $y = (x^2 - 1)(x^2 - 5) = x^4 - 6x^2 + 5$

$\frac{dy}{dx} = 4x^3 - 12x$

$\frac{d^2y}{dx^2} = 12x^2 - 12$

For curve to be concave upwards

$f''(x) > 0$

i.e. $12(x - 1)(x + 1) > 0$

or, $(x - 1)(x + 1) > 0$

i.e. $x < -1$ or $x > 1$

$\Rightarrow |x| > 1$

94.(a) Given circle, $(x - 6)^2 + y^2 = 2$

Equation of tangent is, $Y = mX + a\sqrt{1+m^2}$

Where Y = y, X = x – 6 for this question

or, $y = m(x - 6) + \sqrt{2}\sqrt{1+m^2}$

or, $y = m(x - 6) + \sqrt{2(1+m^2)}$

Focal point of $y^2 = 16x$ is (a, 0) = (4, 0)

Now, focal chord is tangent to circle, so focal point must satisfy equation of tangent so

$0 = m(4 - 6) + \sqrt{2(1+m^2)}$

or, $2m = \sqrt{2(1+m^2)}$

or, $4m^2 = 2 + 2m^2$

or, $m^2 = 1$

or, $m = \pm 1$

95.(c) On, y-axis, x = 0 equation of circle becomes

$y^2 + y - 20 = 0$

$\Rightarrow y = -5$ & 4

So, circle touch y axis at (0, –5) & (0, 4)

Hence, intercept length $\Rightarrow |-5 - 4| = 9$

96.(c) Centroid, $x = \frac{a\cos t + b\sin t + 1}{3}$

$\Rightarrow a\cos t + b\sin t = 3x - 1 \ldots(1)$

$y = \frac{a\sin t - b\cos t}{3}$

$\Rightarrow a\sin t - b\cos t = 3y \ldots (2)$

Squaring & adding (1) & (2)

$a^2\cos^2 t + 2ab\cos t\sin t + b^2\sin^2 t + a^2\sin^2 t - 2ab\cos t\sin t + b^2\cos^2 t$

$= (3x - 1)^2 + (3y)^2$

or, $a^2(\cos^2 + \sin^2 t) + b^2(\sin^2 t + \cos^2 t) = (3x - 1)^2 + (3y)^2$

or, $(3x - 1)^2 + (3y)^2 = a^2 + b^2$

97.(c) 98.(a) 99.(d) 100.(a)

…Best of Luck…