PEA's

TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

B.E. Model Entrance Exam

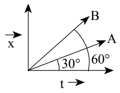
2079

Date: 2079-10-21

Hints and Solutions

Section – I

1.(d) $\frac{v_A}{v_B} = \frac{\tan 30}{\tan 60} = \frac{1}{\sqrt{3} \times \sqrt{3}} = \frac{1}{3}$



2.(b) $E = \frac{1}{2} mu^2$

At highest point $KE' = \frac{1}{2} mu^2 \cos^2 45°$

$= \frac{E}{2}$

3.(a) $R = m(g + a)$ i.e. increases

4.(b) $PE = -\frac{GMm}{r} = -\frac{GMm}{R + R} = -\frac{gR^2 m}{2R}$

$= -\frac{mgR}{2}$

5.(d) $\frac{s'}{s} = \left(\frac{2R}{R}\right)^2 \left(\frac{2T}{T}\right)^4 = 64:1$

6.(b) Heat lost by water = Heat gained by ice

or, $40 \times (90 - \theta) = 40 \times 80 + 40\theta$

or, $3600 - 40\theta = 3200 + 40\theta$

or, $\theta = \frac{400}{80} = 5°C$

7.(b) First case

$f = \frac{v}{2l}$

2nd case

$f' = \frac{v}{4\left(\frac{3l}{4}\right)} = \frac{2f}{3}$

8.(c)

9.(d) $\frac{w_1}{w_2} = \frac{1}{25} = \frac{a_1^2}{a_2^2}$

$\therefore \quad \frac{a_1}{a_2} = \frac{1}{5}$

$\frac{I_{max}}{I_{min}} = \left(\frac{a_1 + a_2}{a_2 - a_1}\right)^2 = \left(\frac{1+5}{5-1}\right)^2 = \frac{9}{4}$

10.(c) $F = E_e = m_p a_p = m_e a_e$

or, $\frac{a_e}{a_p} = \frac{m_p}{m_e}$

11.(c) Charge will not flow if they are at same potential.

12.(b) $\frac{R'}{R} = \left(\frac{l + \frac{l}{10}}{l}\right)^2 = (1.1)^2 = 1.21$

$\therefore \quad R' = 12.1\ \Omega$

13.(c) $\theta_1 - \theta_n = \theta_n - \theta_0$

or, $\theta_n = \frac{\theta_1 + \theta_0}{2} = \frac{530 + 10}{2} = 270°C$

14.(a) $W = MH(\cos 0° - \cos 60°) = MH\left(1 - \frac{1}{2}\right)$

$\therefore \quad MH = 2W$

Again $\tau = MH\sin\theta = 2W \times \frac{\sqrt{3}}{2} = \sqrt{3}\ W$

15.(b)

16.(a) Work function depends on nature of metal.

17.(c) Voltage gain $= \frac{V_{out}}{V_{in}}$

$= \frac{I_c R_{out}}{I_b R_{in}}$

$= \beta \times \frac{5000}{500}$

$= 60 \times 10 = 600$

18.(a) $(A + B).(A - B) = A^2 - B^2$

i.e. $A^2 - AB + BA - B^2 = A^2 - B^2$ which is possible only if $AB = BA$

19.(b) $|(4 + 3i)(x + iy)| = |3 - 4i|$

$\Rightarrow \sqrt{4^2 + 3^2}.\sqrt{x^2 + y^2} = \sqrt{3^2 + (-4)^2}$

$\Rightarrow 5.\sqrt{x^2 + y^2} = 5$

$\Rightarrow x^2 + y^2 = 1$

20.(b) $\lim_{x \to 0} \frac{\sin 4x}{\tan ax} = \lim_{x \to 0} \frac{\frac{\sin 4x}{4x} \times 4}{\frac{\tan ax}{ax}.a}$

$\Rightarrow \frac{4}{a} = 5 \Rightarrow a = \frac{4}{5}$

21.(c) a, b, c are in AP $\Rightarrow 2b = a + c \Rightarrow 3^{2b} = 3^{a+c}$

$\Rightarrow (3^b)^2 = 3^a.3^c \Rightarrow 3^a, 3^b, 3^c$ are in G.P.

22.(a) The product $\sin^7 x \cos^5 x$ is an odd function

So $\int_{-11}^{11} \sin^7 x \cos^5 x\, dx = 0$

23.(d) $y^2 + 2y + x = 0 \Rightarrow y^2 + 2y + 1 = -x + 1$

$\Rightarrow (y + 1)^2 = -(x - 1)$

$\Rightarrow \{y - (-1)\}^2 = 4.\left(-\frac{1}{4}\right).(x - 1)$

$\Rightarrow$ Vertex = (1, –1), lies in 4th quadrant.

24.(d) As $b > a$ i.e. $\frac{1}{a} > \frac{1}{b}$ i.e. $\frac{1}{a} - \frac{1}{b} > 0$

25.(b) We have $(1 + x)^n = C_0 + C_1 x + C_2 x^2 + \ldots + C_n x^n$

Putting $x = 2$, $3^n = C_0 + 2C_1 + 2^2 C_2 + \ldots + 2^n C_n$

26.(c) If the line $lx + my + n = 0$ is normal then it passes through the centre of the circle.

So $l.0 + m.0 + n = 0 \Rightarrow n = 0$

27.(d) Eq^n is $\frac{x^2}{25} + \frac{y^2}{9} = 1$

So length of latus rectum $= \frac{2b^2}{a} = 2.\frac{9}{5} = \frac{18}{5}$

28.(c) By definition $\vec{a} \times \vec{b}$ is perpendicular to both $\vec{a}$ and $\vec{b}$.

29.(b) $\frac{\cos C + \cos A}{c + a} + \frac{\cos B}{b}$

$= \frac{b\cos C + b\cos A + c\cos B + a\cos B}{b(a + c)}$

$= \frac{(a + b)}{b(a + c)} = \frac{1}{b}$

30.(b) For minimum value is $2x - 4 = 0 \Rightarrow x = 2$

31.(c) $3\text{cosec}^2 x - 4 = 0 \Rightarrow \sin^2 x = \frac{3}{4} \Rightarrow \sin^2 x = \left(\frac{\sqrt{3}}{2}\right)^2$

$\Rightarrow \sin^2 x = \sin^2 \frac{\pi}{3} \Rightarrow x = n\pi \pm \frac{\pi}{3}$

32.(b) $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3} \Rightarrow \frac{\pi}{2} - \cos^{-1}x + \frac{\pi}{2} - \cos^{-1}y = \frac{2\pi}{3} \Rightarrow$

$\cos^{-1}x + \cos^{-1}y = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$

33.(c) As the vertices right angled Δ so circumcentre = midpoint of hypotenous

$= \left(\frac{3+0}{2}, \frac{0+4}{2}\right) = \left(\frac{3}{2}, 2\right)$

34.(a) $\sin^2\alpha + \sin^2\beta + \sin^2 r = 1 - \cos^2\alpha + \cos^2\beta + 1 - \cos^2\gamma =$
$3 - (\cos\alpha^2 + \cos^2\beta + \cos^2\gamma) = 3 - 1 = 2$

35.(c) $x^2 - y^2 = \left(\frac{e^t + e^{-t}}{2}\right)^2 + \left(\frac{e^t - e^{-t}}{2}\right)^2 = \frac{1}{4}.4\ e^t.e^{-t} = 1$

36.(b) For roots equal in magnitude but opposite in sign, we must have coefficient of x = 0

$\Rightarrow \quad 5 + k = 0 \quad \Rightarrow k = -5$

37.(a) n!, (n+1)! = (n+1) n!, (n+2)! = (n+2) (n+1)n!
So their HCF = n!

38. (a) 22400 ml of CO = 6.02×10^{23} no. of C atoms

112 ml of CO $= \frac{6.02 \times 10^{23} \times 112}{22400}$

$= 3.01 \times 10^{21}$

39.(c) $200 \times 0.12 = (200 + x) \times 0.1$

$x = \frac{200 \times 0.12}{0.1} - 200 = 40$ ml

40.(b) 4gm He = $2N_A$

1gm He $= \frac{2N_A}{4} = 0.5\ N_A$

41.(c)

42.(a) 44gm of CO_2 at STP = 22.4 litre

4.4gm of STP $= \frac{22.4}{44} \times 4.4 = 2.24$ litre

43.(c) Cu + Zn = Brass
Cu + Sn = Bornze, Cu + Mn = No. any alloy
Cu + Ni = Constant

44.(d) $HgCl_2 + 2KI \rightarrow HgI_2 + 2KCl$
$HgI_2 + 2KI \rightarrow K_2[HgI_4]$
$\quad 2K^- + [HgI_4]^{2-}$

45.(b) $H_2S + (CH_3COO)_2Pb \rightarrow PbS\downarrow + 2CH_3COOH$
black ppt

46.(b) $NH_4Cl + KNO_2 \rightarrow NH_4NO_2 + HCl$
$NH_4NO_2 \rightarrow N_2 + 2H_2O$

47.(c) Na is hightly electropositive metal which is extracted by electrometallry.

48.(b)

49.(b)	50.(c)	51.(b)	52.(c)	53.(c)	54.(c)
55.(a)	56.(a)	57.(c)	58.(a)	59.(c)	60.(b)

Section – II

61.(a) $h = h_1 - h_2$

$= \frac{1}{2} g \times 3^2 - \frac{1}{2} g\,(2)^2$

$= 5 \times 5 = 25$ m

62.(d) R = 3H

or, $\frac{u^2 \sin 2\theta}{g} = \frac{3 \times u^2 \sin^2\theta}{2g}$

or, $2\sin\theta.\cos\theta = \frac{3\sin^2\theta}{2}$

or, $\tan\theta = \frac{4}{3}$

$\theta = \tan^{-1}\left(\frac{4}{3}\right)$

63.(b) Ap. wt of first = Ap. wt of 2nd

$32 - \frac{32}{8} \times \sigma_w = x - \frac{x}{5}\sigma_w$

or, $28 = x\left(\frac{4}{5}\right)$

or, x = 35g

64.(c) $\eta = \left\{1 - \left(\frac{1}{5}\right)^{\gamma-1}\right\} \times 100\%$

$= \left\{1 - \left(\frac{1}{2}\right)^{5/3-1}\right\} \times 100\%$

$= \{1 - (0.5)^{2/3}\} \times 100\%$

= 37%

65.(c) $\frac{t_2}{t_1} = \frac{x_2^2 - x_1^2}{x_1^2 - x_0^2} = \frac{2^2 - 1^2}{1^2 - 0^2}$

or, $t_2 = 3 \times 7 = 21$ hrs.

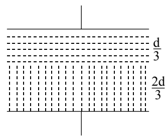
66.(a) $f = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2l'}\sqrt{\frac{T-U}{m}}$

or, $\frac{1}{l}\sqrt{v\rho g} = \frac{1}{l'}\sqrt{v(\rho - \sigma)g}$

or, $\frac{l'}{l} = \sqrt{\frac{\rho - \sigma}{\rho}} = \sqrt{\frac{8-1}{8}}$

$\therefore \quad l' = 0.4\sqrt{\frac{7}{8}} = 0.37$ m

67.(c) $c = \frac{\varepsilon_0 A}{d} = 9$ … (i)



$c_1 = \frac{\varepsilon_0\varepsilon_r A}{\frac{d}{3}} = 3 \times 3 \times 9$

$= 81\ \mu F$

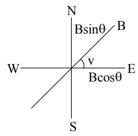
$c_2 = \frac{\varepsilon_0\varepsilon_r' A}{\frac{2d}{3}} = \frac{3}{2} \times 6 \times 9 = 81\ \mu F$

$c_{eq} = \frac{81}{2} = 40.5\ \mu F$

68.(d) $n = \frac{250}{25} = 10$

R = (n – 1) G = (10 – 1) 100 = 900Ω

69.(c) When plane is NS direction



τ_1 = BcosqINA

or, 0.04 = BINAcosθ …(1)

When plane is EW direction

τ_2 = BsinθINA

or, 0.03 = BINAsinθ …(2)

Squaring & adding

$0.04^2 + 0.03^2 = B^2I^2N^2A^2(\cos^2\theta + \sin^2\theta)$

or, $B = \sqrt{\frac{0.04^2 + 0.03^2}{2^2 \times 50^2 \times (1.25 \times 10^{-3})^2}} = 0.4T$

70.(a) $I = \frac{E}{R} = -\left(\frac{d\phi}{dt}\right)\frac{1}{R}$
$= -\frac{(12t-5)}{10}$
$= -\frac{(12 \times 0.25 - 5)}{10} = 0.2A$

71.(b) For near object
$u = ?\ v = -25\text{ cm } f = \frac{1}{p} = 25\text{ cm}$
$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$
or, $\frac{1}{25} = \frac{1}{u} - \frac{1}{25}$
or, $\frac{1}{u} = \frac{2}{25} \Rightarrow u = 12.5\text{ cm}$
For distant object
$u = ?\quad v = \infty, f = 25\text{ cm}$
$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$
or, $\frac{1}{25} = \frac{1}{u} + \frac{1}{\infty}$
or, $u = 25\text{ cm}$
$\therefore$ Range = 12.5 cm to 25 cm

72.(c) $\beta = \frac{3D \times 3\lambda}{\frac{d}{3}} = \frac{27\,D\lambda}{d}$
No of fringes (n) $= \frac{\frac{1}{3}}{\beta} = \frac{1 \times d}{3 \times 27D\lambda} = \frac{d}{81D\lambda}$

73.(a) $E = \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{975 \times 10^{-10}} = 12.73\text{ eV}$
$E_f = E_1 + E = -13.6 + 12.73 = -0.87\text{ eV} = -\frac{13.6}{n^2}$
$\therefore\quad n = 4$
$E_4 - E_3 = \frac{hc}{\lambda}$
$\lambda = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{(-0.85 + 1.51) \times 1.6 \times 10^{-19}}$
$= 18806 \times 10^{-10}\text{ m} = 18806\text{ Å}$

74.(d) $m = m_0\left(\frac{1}{2}\right)^{t/T_{1/2}} = m_0'\left(\frac{1}{2}\right)^{t/T_{1/2}}$
or, $40\left(\frac{1}{2}\right)^{t/20} = 160\left(\frac{1}{2}\right)^{t/10}$
or, $\left(\frac{1}{2}\right)^{t/20 + 2} = \left(\frac{1}{2}\right)^{t/10}$
or, $\frac{t}{20} + 2 = \frac{t}{10}$
or, $\frac{t}{10} - \frac{t}{20} = 2$
or, $t = 40s$

75.(d) $\int\frac{x^4}{x + x^5}dx = \int\frac{1 + x^4 - 1}{x(1 + x^4)}dx$
$= \int\frac{1}{x}dx - \int\frac{dx}{x(1 + x^4)}$
$= \log_e x - f(x) - c' + k = \log_e x - f(x) + c$

76.(c) $y = c_1e^{nx} + c_2e^{-nx}$, So $\frac{dy}{dx} = nc_1e^{nx} - nc_2e^{-nx}$, $\frac{d^2y}{dx^2} = n^2c_1e^{nx} + n^2c_2e^{-nx}$
$= n^2(c_1e^{nx} + c_2e^{-nx}) = n^2y$

77.(d) $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$
$\Rightarrow\quad \sin^{-1}(x\sqrt{1 - y^2} + y\sqrt{1 - x^2}) = \pi - \sin^{-1}z$
$\Rightarrow\quad x\sqrt{1 - y^2} + y\sqrt{1 - x^2} = \sin(\pi - \sin^{-1}z) = \sin\sin^{-1}z = z$

78.(a) A, B, C are in AP $\Rightarrow 2B = A + C \Rightarrow B = 60°$,
a, b, c are in GP $\Rightarrow b^2 = ac$
So, $\cos 60° = \frac{c^2 + a^2 - b^2}{2ca} \Rightarrow \frac{1}{2} = \frac{c^2 + a^2 - b^2}{2b^2}$
$\Rightarrow\quad b^2 = c^2 + a^2 - b^2$
$\Rightarrow\quad 2b^2 = c^2 + a^2$
$\Rightarrow\quad a^2, b^2, c^2$ are in A.P.

79.(c) $x + iy = k + 3 + i\sqrt{5 - k^2} \Rightarrow x = k + 3$,
$y = \sqrt{5 - k^2}$
$\Rightarrow\quad (x - 3)^2 = k^2, y^2 = 5 - k^2$
$\Rightarrow\quad (x - 3)^2 + y^2 = 5$, a circle

80.(d) α and β are the roots of $x^2 + px + q = 0$
α' and β' are the roots of $x^2 + 9x + p = 0$
So, $\alpha + \beta = -p, \alpha\beta = q$
So $\alpha' + \beta' = -q, \alpha'\beta' = p$
Now, $\alpha - \beta = \alpha' - \beta'$
$\Rightarrow\quad (\alpha + \beta)^2 - 4\alpha\beta = (\alpha' + \beta')^2 - 4\alpha'\beta'$
$\Rightarrow\quad p^2 - 4q = q^2 - 4p$
$\Rightarrow\quad (p^2 - q^2) + 4(p - q) = 0$
$\Rightarrow\quad (p - q)(p + q + 4) = 0$
$\Rightarrow\quad p + q = -4$

81.(b) $x = a + \frac{a}{r} + \frac{a}{r^2} + \ldots$
$\Rightarrow x = a\left(1 + \frac{1}{r} + \frac{1}{r^2} + \ldots\right) = a\frac{1}{1 - \frac{1}{r}} = a.\frac{r}{r - 1}$
$y = b - \frac{b}{r} + \frac{b}{r^2}\ldots \Rightarrow y = b\left(1 - \frac{1}{r} + \frac{1}{r^2} - \ldots\right)$
$= b.\frac{1}{1 + \frac{1}{r}} = b.\frac{r}{r + 1}$
$z = c + \frac{c}{r^2} + \frac{c}{r^4} + \ldots \Rightarrow z = c\left(1 + \frac{1}{r^2} + \frac{1}{r^4} + \ldots\right)$
$= c.\frac{1}{1 - \frac{1}{r^2}} = c.\frac{r^2}{r^2 - 1}$
So, $\frac{xy}{z} = \frac{ab\frac{r^2}{r^2 - 1}}{c\frac{r^2}{r^2 - 1}} = \frac{ab}{c}$

82.(d) The function is not defined if $3 - \cos 2x = 0$
$\Rightarrow\quad \cos 2x = 3$, impossible.
So domain = R. Also, $-1 \le \cos 2x \le 1$
So range $= \left[\frac{1}{3 - (-1)}, \frac{1}{3 - 1}\right] = \left[\frac{1}{4}, \frac{1}{2}\right]$

83.(c) $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1, \qquad \vec{a} + \vec{b} + \vec{c} = 0$

$\Rightarrow \quad \vec{a} + \vec{b} = -\vec{c} \qquad \Rightarrow (\vec{a} + \vec{b})^2 = (-\vec{c})^2$

$\Rightarrow \quad |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b} = |\vec{c}|^2$

$\Rightarrow \quad 1 + 1 + 2|\vec{a}|.|\vec{b}| \cos\theta = 1$

$\Rightarrow \quad 2\cos\theta = -1 \quad \Rightarrow \cos\theta = -\frac{1}{2} \Rightarrow \theta = 120°$

84.(c) $y = x - \frac{x^2}{2!} + \frac{y^3}{3!} - \frac{x^4}{4} + \dots \Rightarrow y = \log_e(1 + x)$

$\Rightarrow \quad 1 + x = e^y \Rightarrow x = e^y - 1$

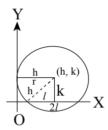
Also, $\frac{y}{1!} + \frac{y^2}{2!} + \frac{y^3}{3!} + \dots = e^y - 1 = x$

85.(a) Given eq^n is $x^2 + y^2 + 2gx + 2fy + 1 = 0$
On comparison, we have
$a = 1, b = 1, h = 0, g = g, f = f, c = 1$
So condition to represent a pair of line is $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$\Rightarrow \quad 1.1.1 + 0 - 1.f^2 - 1.g^2 - 1.0 = 0$

$\Rightarrow \quad 1 = f^2 + g^2$

86.(b) Let (b, k) be centre. As it touches y-axis
So r = h. Then from figure, we have



$h^2 = k^2 + l^2$. So locus is $x^2 = y^2 + l^2$
i.e. $\quad x^2 - y^2 = l^2$

87.(c) Eq^n of planes are $2x - 2y + z + 1 = 0$
i.e. $\quad 4x - 4y + 2z + 2$
$= 0$ and $4x - 4y + 2z + 3 = 0$

So the distance between them $= \frac{3 - 2}{\sqrt{4^2 + (-4)^2 + 2^2}} = \frac{1}{\sqrt{36}} = \frac{1}{6}$

88.(b) Solving the curves $y = x$ and $y = x^3$,
we get $x = 0, 1, -1$

$\therefore \quad$ Required area $= \int_0^1 (x - x^3)\, dx = \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1$

$= \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$ sq. units

89.(c) Let a be the length of the cube. Then

$\frac{dv}{dt} = K$

$\Rightarrow \quad \frac{da^3}{dt} = K \Rightarrow 3a^2\frac{da}{dt} = K \Rightarrow \frac{da}{dt} = \frac{K}{3a^2}$

Also, we have, $s = 6a^2$

$\therefore \quad \frac{ds}{dt} = 6\frac{da^2}{dt} = 6.2a\frac{da}{dt} = 12a.\frac{K}{3a^2} = \frac{4K}{a}$

i.e. $\quad \frac{ds}{dt} \propto \frac{1}{a}$

90.(d) An alkene 'x' ozonolysis gave acetone acid acetaldehyde. The product formed by reacting x with HBr in presence of peroxide is 2-bromopentane.

91.(c) $W = Zit$

$W = \frac{E}{F} \times I \times t$

$2 = \frac{108}{96500} \times 2 \times 35 \times 60$

$I = \frac{2 \times 96500}{108 \times 35 \times 60} = \frac{193000}{226800} = 0.85$ A

$= 0.85 \times 100 = 85\%$

92.(a) $10 \times N_1 = 8 \times 0.625$

$N_1 = \frac{8 \times 0.625}{10} = 0.5$ N

$\frac{W}{E} = \frac{N_r}{1000}$

$\frac{3.55}{E} = \frac{0.5 \times 100}{1000}$

$E = 71$

E or $M_2CO_3.xH_2O = 71$

$2 \times 23 + 60 + 18x = 2 \times 71$

$x = \frac{142 - 106}{18} = \frac{36}{18} = 2$

93.(c) $CaF_2 \rightarrow Ca^{++} + 2F^{-}$

$K_{sp} \qquad x \qquad 2x$

$x \times (2x)^2 = K_{sp}$

$x \times 4x^2 = 3.95 \times 10^{-11}$

$4x^3 = 3.95 \times 10^{-11}$

$x = \sqrt[3]{\frac{3.96 \times 10^{-11}}{4}} = 2.145 \times 10^{-4}$

$[F^{-}] = 2x = 2 \times 2.145 \times 10^{-4} = 4.29 \times 10^{-4}$

94.(d) $4FeS_2 + 11O_2 \rightarrow 2Fe_2O_3 + SO_2$

SO_2 acts axidand, reductant as well as bleachant.

95.(a) $\frac{3}{4}$ of 100 ml = 75 ml

$V_{mix} M_{mix} = V_aM_a - V_bM_b$

$(100 + 75) \times M_{mix} = 100 \times 0.1 - 75 \times 0.1$

$M_{mix} = \frac{10 - 75}{175}$

$M_{mix} = 0.0142857$ M of HCl

$pH = -\log[H^+] = -\log[0.01428587] = 1.84$

96.(b)

97.(b) 98.(b) 99.(c) 100.(d)

…Best of Luck…