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2083-4-02 Hints & Solution

Section – I

1.(b) $R = \sqrt{P^2 + 2P^2\cos\theta + P^2} = \sqrt{2P^2(1+\cos\theta)}$

$= \sqrt{2P^2 \times 2\cos^2\frac{\theta}{2}} = 2P\cos\frac{\theta}{2}$

2.(c) $F = \frac{nmv}{t} = 10 \times 10 \times 10^{-3} \times 500 = 50$ N

3.(b) $\frac{mv^2}{r} = \mu mg \Rightarrow r = \frac{v^2}{\mu g} = \frac{12^2}{0.4 \times 10} = 36$ m

4.(c) $Fs = \frac{1}{2}m(v_1^2 - v_2^2) \Rightarrow F = 75$N

5.(c) $\frac{\Delta v}{v} = 0.12\%$

$\gamma\Delta\theta = 0.12\%$

$3\alpha\Delta\theta = 0.12\%$

$\therefore \quad \alpha = 2 \times 10^{-5}$ /°C

6.(a) $\beta_w = \frac{\beta_a}{\mu_w} = \frac{0.4}{\frac{4}{3}} = 0.3$ mm

7.(d) $I_R = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi$

For maxima, $\cos\phi = 1$

$\Rightarrow \quad I_R = I + 9I + 2\sqrt{9I^2} = 16I$

8.(b) $q = ne = 10^{15} \times 1.6 \times 10^{-19}$ C $= 1.6 \times 10^{-4}$ C

9.(d) $\rho = \frac{RA}{L}$, depends upon nature of materials

10.(a) $IgG = (I - I_g)S$

2% of IG = 98% of IS

or, $\quad S = \frac{G}{49}$

11.(a) Magnetic flux $\phi = LI = 5 \times 10^{-3} \times 2 = 0.01$ weber

12.(c) x-ray are energetic having high penetrating power.

13.(d) $\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] \Rightarrow \frac{1}{\lambda} = \frac{5R}{36}$

$\therefore \quad \lambda = \frac{36}{5R}$

14.(c) $\lambda = \frac{0.693}{t_{1/2}} \Rightarrow \frac{1}{\lambda} = \frac{t_{1/2}}{0.693}$

i.e. $t_m = \frac{1}{\lambda} = 14.43$ yr

15.c	**16.b**	**17.b**	**18.a**	**19.b**	**20.a**	**21.b**
22.c	**23.b**	**24.b**	**25.c**	**26.d**	**27.d**	**28.b**

29.(c) $A \cap B = \phi$ [$y = e^x$ and $y = x$ do not meet for any $x \in \Re$]

30.(c)

31.(c) $x^2 = -\frac{b}{a}$ gives real roots if $\frac{b}{a} < 0 \Rightarrow ab < 0$

32.(b) $\sum_{n=0}^{\infty} \frac{(\ln x)^n}{n!} = e^{\ln x} = x$

33.(a) $\tan 3\theta = \tan(\frac{\pi}{2} - \theta)$

$3\theta = n\pi + \frac{\pi}{2} - \theta \Rightarrow \theta = (2n+1)\frac{\pi}{8}$

34.(c) $\cos^{-1}[\cos(\pi - \frac{\pi}{3})] = \frac{2\pi}{3}$

35.(b) PQ = PR is possible if P^{-1} exists $\Rightarrow$ P is non - singular matrix

36.(a) $T_n = S_n - S_{n-1} = [2n^2 + 5n] - [2(n-1)^2 + 5(n-1)] = 4n + 3$

37.(b) $2x - y + k = 0$ passes through (-3, 3)

So $-6-3+k = 0$

$\therefore$ k = 9

38.(c) Length $= \sqrt{1 + 4 + 9} = \sqrt{14}$

39.(c) $\frac{(x-3)^2}{9} + \frac{(y-4)^2}{25} = 1$ so ellipse

40.(c) The point is on directrix so angle between tangent is 90°

41.(d) For $x < 3$, $\sqrt{x-3}$ is undefined so does not exist.

42.(b) Valid $|\frac{3x}{6}| < 1 \Rightarrow |x| < 2$

43.(a) $\frac{dx}{dy} = \frac{2t}{2t-1} = 0, \qquad \therefore t = 0$

44.(b) $\int a^3.a^{3x}\,dx = a^3\frac{a^{3x}}{3\log a} + c = \frac{a^{3x+3}}{3\log a} + c$

45.(c) $|4\vec{a} + 3\vec{b}|^2 = 16 \times 9 + 9 \times 16 + 2.4.3.3.4\left(-\frac{1}{2}\right)$

$= 144 + 144 - 144 = 144$

$|4\vec{a} + 3\vec{b}| = 12$

46.(d) If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, their graph is intersecting.

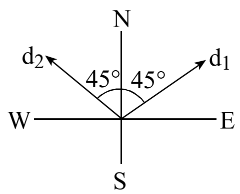
47.(b) $\sum_{r=1}^{n} \frac{^nP_r}{r!} = \sum_{r=1}^{n} {^nC_r} = {^nC_1} + {^nC_2} + \ldots.. {^nC_n} = 2^n - 1$

48.(c)

49.(a)	**50.(c)**	**51.(b)**	**52.(b)**	**53.(c)**	**54.(d)**
55.(c)	**56.(b)**	**57.(a)**	**58.(c)**	**59.(b)**	**60.(b)**

Section – II

61.(c)



$dx = d_1\sin45°\hat{\imath} + d_2\sin45°(-\hat{\imath}) = \sqrt{2}\,\hat{\imath}$

$dy = d_1\cos45°\hat{\jmath} + d_2\cos45°(\hat{\jmath}) = 5\sqrt{2}\,\hat{\jmath}$

$d = \sqrt{d_x^2 + d_y^2} = \sqrt{\sqrt{2}^2 + (5\sqrt{2})^2} = 2\sqrt{13}$ km

62.(b) $\tau = I\alpha,\ \alpha = \frac{\tau}{I} = \frac{200}{100} = 2$ rad/s^2

$\omega^2 = \omega_0^2 + 2\alpha\theta = 160\pi$

$K.E. = \frac{1}{2}I\omega^2 = 25{,}133J$

63.(b) $\frac{V}{t} = \frac{\pi Pr^4}{8\eta}$

i.e. $P = \frac{v}{t} \times \frac{8\eta}{\pi r^4} = 6.5\times10^4$ N/m^2

64.(c) From 1st and 2nd : $\frac{(\frac{d\theta}{dt})_1}{(\frac{d\theta}{dt})_2} = \frac{\theta_1-\theta_0}{\theta_2-\theta_0}$

i.e. $\theta_0 = 35°C$

From 1st and 3rd: $\frac{(\frac{d\theta}{dt})_1}{(\frac{d\theta}{dt})_3} = \frac{\theta_1-\theta_0}{\theta_3-\theta_0}$

$\therefore$ t = 15 min

65.(b) The distance of 3rd dark and 5th bright fringe will be 2.5β.

$\therefore\ 2.5\beta = 2.5\times\frac{\lambda D}{d} = \frac{2.5\times650\times10^{-9}\times1}{10^{-3}} = 1.62$ mm

66.(d) $f = \frac{1}{2L}\sqrt{\frac{T}{\mu}},\ \mu = \frac{m}{L} = \frac{0.01}{0.5} = 0.02$

T = 800 N, i.e. $f = \frac{1}{2\times0.5}\sqrt{\frac{800}{0.02}} = 200$ Hz

67.(d) $P.E._1 = \frac{q_1q_2}{4\pi\varepsilon_0 r_1}$, $r_1 = 12$ cm

$P.E._1 = \frac{q_1q_2}{4\pi\varepsilon_0 r_2}$, $r_2 = 12 - 4 = 8$ cm

$W = \Delta P.E. = P.E._2 - P.E._1$

$W = \frac{q_1q_2}{4\pi\varepsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = 1.875 \times 10^{12}$ J

68.(b) Let x be the value of resistance to be connected in series with the galvanometer

Then, $V = I_g(X + G)$

i.e. $X = 75\Omega$

69.(b) $\tau = BINA\sin\theta,\ \theta = 90°$

$\tau = BINA = 4 \times 10^{-3}$ Nm

70.(b) $I = \frac{V_R}{R} = \frac{20}{40} = 0.5A$

$Z = \frac{50}{0.5} = 100\Omega$

$Z = \sqrt{(R+r)^2 + X_L^2};\ X_L = 2\pi fl$

$\therefore\ r = 37.7\Omega$

71.(c) $F_eF_m, eE = BeV$

$\frac{V}{d} = Bv \Rightarrow V = Bvd = 5\times10^{-2}\times5\times10^{6}\times10^{-2} = 2500$ volt.

72.(b) $V = 20kV = 20\times10^3$ volt

$eV = hf_{max}$

$eV = \frac{hc}{\lambda_{min}} \Rightarrow \lambda = \frac{hc}{eV} = 0.62Å$

73.(b) $\bar{B} = \frac{B.E.}{A} = \frac{[Zm_p + (A-Z)m_n - M]\times931}{7} = 5.6$ MeV

74.(a) $MnO_4^- + 8H^+ + 5e^- \rightarrow Mn^{2+} + 4H_2O]\times3$

$Fe^{2+} \rightarrow Fe^{3+} + 1e^-]\times5$

As 5 moles of $Fe(C_2O_4)$ = 3 moles of $KmnO_4$

So, 1 mol of $Fe(C_2O_4)$ = 3/5 moles of $KmnO_4$ = 0.6 mol

75.(a) $N_{mix} = (N_1V_1 + N_2V_2 + N_3V_3)/V_{total}$

76.(b) 71 parts of chlorine combine with 32 parts sulphur 35.5 parts of chlorine combine with 16 parts of sulphur

Hence, eq.wt of S in $SCl_2 = 16$

77.(d) 1mol of Au = 197g = 0.197kg = 6.02×10^{23} atoms so, 19.7 kg Au = 6.02×10^{25} atoms

78.(a) No. of mol × N_A

79.(c) Bond length order: Single bond > bond created by resonance > double bond > triple bond

80.(c)

81.(c) B shows + I effect and hyperconjugation

C shows - I effect D shows -R and -I effect

82.(a) Here $g(x) = \log_e(x + \sqrt{x^2 + 1})$

$g(-x) = \log_e(-x+\sqrt{x^2 + 1}) = -\log_e(x + \sqrt{x^2 + 1}) = -g(x)$

So g(x) is odd. Also sine is odd. So their composition is odd.

83.(d) Given $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \frac{1}{5^4} + \frac{1}{6^4} + \ldots \text{ to } \infty = \frac{\pi^4}{90}$

or, $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots + \left(\frac{1}{2^4} + \frac{1}{4^4} + \frac{1}{6^4} + \ldots\right) = \frac{\pi^4}{90}$

or, $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots + \left(\frac{1}{2^4\,1^4} + \frac{1}{2^4\,2^4} + \frac{1}{2^4\,3^4} + \ldots\right) = \frac{\pi^4}{90}$

or, $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots + \frac{1}{2^4}\left(\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \ldots\right) = \frac{\pi^4}{90}$

or, $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots + \frac{1}{16}\frac{\pi^4}{90} = \frac{\pi^4}{90}$

or, $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots = \frac{\pi^4}{90} - \frac{1}{16}\frac{\pi^4}{90} = \frac{15}{16}\frac{\pi^4}{90}$

$= \frac{\pi^4}{96}$

84.(b) Given $\left(\frac{1-i}{1+i}\right)^{100} = a + ib$

or, $\left(\frac{1-i}{1+i} \times \frac{1-i}{1-i}\right)^{100} = a + ib$

or, $\left(\frac{1-2i+i^2}{1^2-i^2}\right)^{100} = a + ib$

or, $\left(\frac{1-2i-1}{1+1}\right)^{100} = a + ib$

or, $\left(-\frac{2i}{2}\right)^{100} = a + ib$

or, $(-i)^{100} = a + ib$

$(i)^{100} = a + ib$

or, $(i^4)^{25} = a + ib$

or, $1 = a + ib$

or, $1 + 0i = a + ib$

$\Rightarrow$ $a = 1, b = 0$

85.(c) Given $\begin{vmatrix} a & 1 & a+c \\ b & 1 & c+a \\ c & 1 & a+b \end{vmatrix}$

Applying $C_3 \to C_3 + C_1$, $\begin{vmatrix} a & 1 & a+b+c \\ b & 1 & a+b+c \\ c & 1 & a+b+c \end{vmatrix}$

$= (a + b + c)\begin{vmatrix} a & 1 & 1 \\ b & 1 & 1 \\ c & 1 & 1 \end{vmatrix} = 0\ (\because C_2 = C_3)$

86.(b) $C(n, 2) = 28$

or, $\frac{n!}{(n-2)!\,2!} = 28$

or, $\frac{n(n-1)\,(n-2)!}{(x-2)!\,2!} = 28$

or, $x^2 - n - 56 = 0$

or, $(n - 8)(n + 7) = 0$

$\therefore$ $n = 8$

87.(b) Given $(1 + x^2)^{40}\left(x^2 + 2 + \frac{1}{x^2}\right)^{-5}$

$= (1 + x^2)^{40}\left\{\left(x + \frac{1}{x}\right)^2\right\}^{-5}$

$= (1 + x^2)^{40}\left(\frac{1+x^2}{x}\right)^{-10}$

$= (1 + x^2)^{40}\frac{(1+x^2)^{-10}}{x^{-10}} = (1 + x^2)^{30}\,x^{10}$

So, coefficient of x^{20} in $(1 + x^2)^{30}\,x^{10}$

= Coefficient of x^{10} in $(1 + x^2)^{30}$

= C(30, 5) or C(30, 25)

88.(b) Let $y = m_1x$ and $y = m_2x$ be the lines represented by $x^2 + \lambda xy - 3y^2 = 0$. Then $m_1 + m_2 = \frac{\lambda}{-3}$, $m_1m_2 = \frac{1}{-3}$

Here, $m_1 + m_2 = 2m_1m_2$

or, $-\frac{\lambda}{3} = 2 \times -\frac{1}{3}$

$\Rightarrow$ $\lambda = 2$

89.(c) Eccentricity of hyperbola

$= \sqrt{1 + \frac{81}{144}} = \sqrt{\frac{225}{144}} = \frac{15}{12} = \frac{5}{4}$

and foic = $(\pm ae, 0) = \left(\pm \frac{12}{5} \times \frac{5}{4}, 0\right) = (\pm 3, 0)$

Again foci of ellipse = $(\pm ae, 0) = (\pm\sqrt{16 - b^2}, 0)$

So, $\sqrt{16 - b^2} = 3 \Rightarrow 16 - b^2 = 9 \Rightarrow b^2 = 7$

90.(b) $|\vec{a} \times \vec{i}|^2 + |\vec{a}|\,|\vec{i}| \sin\alpha = |\vec{a}| \sin\alpha$

$|\vec{a} \times \vec{i}|^2 = |\vec{a}|^2 \sin^2\alpha$

$|\vec{a} \times \vec{j}|^2 = |\vec{a}|^2 \sin\beta$

$|\vec{a} \times \vec{k}|^2 = |\vec{a}|^2 \sin^2\gamma$

So, $|\vec{a} \times \vec{i}|^2 + |\vec{a} \times \vec{j}|^2 + |\vec{a} \times \vec{k}|^2$

$= |\vec{a}|^2 (\sin^2\alpha + \sin^2\beta + \sin^2\gamma) = 2|\vec{a}|^2$

[Note that line makes angles α, β, γ with the coordinate axes].

91.(d) Given $\cos^{-1}\sqrt{p} + \cos^{-1}\sqrt{1-p} + \cos^{-1}\sqrt{1-q} = \frac{3\pi}{4}$

or, $\cos^{-1}\sqrt{p} + \sin^{-1}\sqrt{p} + \cos^{-1}\sqrt{1-q} = \frac{3\pi}{4}$

or, $\cos^{-1}\sqrt{1-q} = \frac{\pi}{4}$

or, $\sqrt{1-q} = \cos\frac{\pi}{4}$

$\Rightarrow$ $\sqrt{1-q} = \frac{1}{\sqrt{2}}$

$\Rightarrow \quad 1 - q = \frac{1}{2} \qquad \Rightarrow \quad q = \frac{1}{2}$

92.(c) Given $f(x) = x^n$, $f'(x) = nx^{n-1}$, $f''(x) = n(n-1)x^{n-2}$, ..., $f^n(x) = n(n-1)(n-1) .... 1 = n!$

Now, $f(1) - \frac{f'(1)}{1!} + \frac{f''(1)}{2!} - ... + \frac{(-1)^n f^n(1)}{x!}$

$= 1 - \frac{n}{1!} + \frac{n(n-1)}{2!} - .... + (-1)^n \frac{n!}{n!}$

$= C(n, 0) - C(n, 1) + C(n, 2) - ....+(-1)^n C(n,n)$

$= (1-1)^n = 0$

93.(a) Given, $y^2 = 5x - 1$

or, $\frac{dy^2}{dy}\frac{dy}{dx} = 5\frac{dx}{dx} - \frac{d}{dx}$

or, $2y\frac{dy}{dx} = 5$

or, $\frac{dy}{dx} = \frac{5}{2y}$

At point $(1, -2)$, slope of tangent $= \frac{5}{2x-2} = -\frac{5}{4}$

So, slope of normal $= \frac{4}{5}$

Normal is $ax - 5y + b = 0$

So, slope $= -\frac{a}{-5}$

or, $\frac{4}{5} = \frac{a}{5}$

i.e. $a = 4$

Also, $4 \times 1 - 5x - 2 + b = 0$

$4 + 10 + b = 0$

i.e. $b = -14$

94.(b) $I = \int \frac{x \sin^{-1}x}{\sqrt{1-x^2}} dx$

Put $y = \sin^{-1}x$

$\therefore \quad dy = \frac{1}{\sqrt{1-x^2}} dx$

Then $I = \int y\sin y \, dy$

$= y\int \sin y \, dy - \int \left(\frac{dy}{dx}\int \sin y \, dy\right) dy$

$= -y\cos y - \int -\cos y \, dy$

$= -y\cos y + \sin y + c = -\sin^{-1}x\sqrt{1-x^2} + x + c$

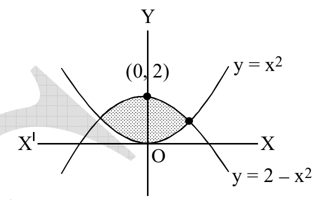
95.(b) $\int_{-1}^{3} \left[\tan^{-1}\frac{x}{x^2+1} + \tan^{-1}\frac{x^2+1}{x}\right] dx$

$= \int_{-1}^{3} \left[\tan^{-1}\frac{x}{x^2+1} + \tan^{-1}\frac{x}{x^2+1}\right] dx$

$= \int_{-1}^{3} \frac{\pi}{2} dx = \frac{\pi}{2}\int_{-1}^{3} dx = \frac{\pi}{2}[x]_{-1}^{3} = \frac{\pi}{2}[3-(-1)]$

$= \frac{\pi}{2} \times 4 = 2\pi$

96.(a)



Given curves are

$y = x^2$ .... (i) $\qquad y = 2 - x^2$ ... (ii)

Solving (i) and (ii),

$x^2 = 2 - x^2$

or, $2x^2 = 2$ or, $x^2 = 1$ i.e., $x = \pm 1$

Thus the required area

$= 2\int_a^b (y_1 - y_2)\, dx$

$= 2\int_0^1 (2 - x^2 - x^2)\, dx$

$= 2\int_0^1 (2 - 2x^2)\, dx$

$= 4\int_0^1 (1 - x^2)\, dx$

$= 4\left[x - \frac{x^3}{3}\right]_0^1 = 4\left(1 - \frac{1}{3}\right) - 0$

$= 4 \times \frac{2}{3} = \frac{8}{3}$ sq. units

97.(d) **98.(a)** **99.(c)** **100.(d)**

…The End…