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2083-04-01 Hints & Solution

Section – I

1.(d) $Bx = Dt$

or, $\frac{D}{B} = LT^{-1}$

2.(b) $R = \frac{u^2 \sin 2\theta}{g}$, R will be max if

$\sin 2\theta = 1 = \sin 90°$

$\theta = 45°$

3.(c) $\frac{1}{2} m (2v_e)^2 - \frac{1}{2} mv_e^2 = \frac{1}{2} mv'^2$

or, $\sqrt{3}\, v_e = v'$

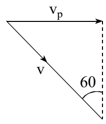
4.(c) $r = \sqrt{r_1^2 + r_2^2} = \sqrt{3^2 + 4^2} = 5$ cm

5.(c) i.e. 0 to 4°C → volume decreases, 4°C to 15°C → volume increases

6.(c) $a_2 = 6$ unit, $a_1 = 8$ unit

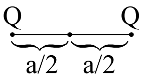
$\frac{I_{max}}{I_{min}} = \left(\frac{a_1+a_2}{a_1-a_2}\right)^2 = \left(\frac{8+6}{8-6}\right)^2 = \left(\frac{14}{2}\right)^2 = 49 : 1$

7.(c) $\sin 60° = \frac{v_p}{v}$



$v_p = \frac{\sqrt{3}v}{2}$

8.(c)



$\frac{Qq}{4\pi\varepsilon_0 \frac{a^2}{4}} = \frac{Q.Q}{4\pi\varepsilon_0 a^2} \Rightarrow q = \frac{Q}{4}$

9.(b) $\frac{R'}{R} = \left(\frac{2l}{l}\right)^2 = 4$

$R' = 4R$

10.(c) $E = \frac{\Delta\phi}{\Delta t} = \frac{8\times10^{-4}}{0.5} = 1.6$mV

11.(a) $V_L = 60V$, $V_C = 30V$, $V_R = 40V$

$V = \sqrt{V_R^2 + (V_L - V_C)^2} = 50V$

12.(d) $v = \sqrt{\frac{2eV}{m}} = \sqrt{2\times1.8\times10^{11}\times100} = 6\times10^6$ m/s

13.(a) $\phi = hf_0 \Rightarrow f_0 = \frac{\phi_0}{h} = 8\times10^{14}$ Hz

14.(b) P-type semiconductor holes are the majority charge carriers.

15.(c) de Broglie wavelength $\lambda = \frac{h}{mv}$; m is maximum for α-particle

16.(a) CH_3^+ possess sp^2-hybridisation

17.(d) CrO_2Cl_2

18.(a) The extent of H-bonding is maximum in glycerol due to three –OH groups per molecule.

19.(a)

20.(c) Two miscible liquids can be separated by distillation

21.(a)

22.(a)

23.(a)

24.(b)

25.(a) $CaCO_3 + SiO_2 \rightarrow CaSiO_3 + CO_2$

Flux impurity slag

26.(c)

27.(d)

28.(c)

29.(b) Here, $f(-x) = f(x) \rightarrow$ even

$f(-x) = -g(x) \rightarrow$ odd

Now, $(fog)(-x) = f(g(-x)) = f(-g(x)) = f(g(x))$

$= (fog)(x)$

So, composite of even and odd functions is even.

30.(c) $A^2 - A + I = 0$ or $I = A - A^2$ or $I = A(I - A)$

$\therefore \quad A^{-1} = I - A$

31.(c) We have, Adj. $A = |A|\, I = 3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

32.(b) Given $x = 0 \rightarrow$ y-axis

$y = 0 \rightarrow$ x-axis and line $4x + 5y = 20$

Put $x = 0$, $y = 4$ which is y-intercept

Put $y = 0$ x $x = 5$ which is x-intercept

Now, area of triangle $= \frac{1}{2} \times$ x-intercept × y-intercept

$= \frac{1}{2} \times 5 \times 4 = 10$ sq. units

33.(c) The circle is $x^2 + y^2 = 16$

Now, $5^2 + 4^2 - 16 = 25 > 0$. So the point (5, 4) is external and 2 tangents are possible.

34.(b) The parametric equations are

$x = a\,\text{sech}\, t$, $y = b$ canh t

Now, $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = \text{sech}^2 t + \tanh^2 t$

i.e., $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, which is an ellipse.

35.(d) Given conic section is $4x^2 - 9y^2 = 36$

i.e., $\frac{x^2}{9} - \frac{y^2}{4} = 1$ which is a hyperbola and a = 3, b = 2. So $|PS - PS'| = 2a = 2 \times 3 = 6$.

36.(c) Given equation is $x^2 + y^2 = 0$ which implies

$x = 0, y = 0$

Here we can take any value of Z. So (0, 0, Z) lies on Z-axis.

37.(b) Here, centroid of triangle ABC = (2, 3, 4)

So, coordinates of A = (3 × 2, 0, 0) = (6, 0, 0) ⇒ x-intercept = 6

Coordinates of B = $(0 \times 3 \times 3, 0) = (0, 9, 0) \Rightarrow$ y-intercept = 9
Coordinates of C = $(0, 0, 3 \times 4) = (0, 0, 12) \Rightarrow$ z-intercept = 12
$\therefore$ Required equation of plane is
$\frac{x}{6} + \frac{y}{9} + \frac{z}{12} = 1$ or, $\frac{6x + 4y + 3z}{36} = 1$
i.e., $6x + 4y + 3z = 36$

38.(b) Here, $\vec{a} = \vec{i} + 2\vec{j} + 2\vec{k}$ So, $|\vec{a}| = \sqrt{1^2 + 2^2 + 2^2} = 3$
$\vec{b} = 6\vec{i} - 2\vec{j} + 3\vec{k}$ So, $|\vec{b}| = \sqrt{6^2 + (-2)^2 + 3^2} = 7$
$\therefore \frac{\text{Projection of } \vec{b} \text{ on } \vec{a}}{\text{Projection of } \vec{a} \text{ on } \vec{b}} = \frac{|\vec{b}|}{|\vec{a}|} = \frac{7}{3}$

39.(c) Given a = 1, b = 2, C = 60°
We have, area of $\Delta ABC = \frac{1}{2} \times ab \sin C$
$= \frac{1}{2} \times 1 \times 2 \times \sin 60° = \frac{\sqrt{3}}{2}$ sq. units

40.(a) Regression equations are
$20x - 9y - 107 = 0$ and $4x - 5y + 33 = 0$
The point of intersection (13, 17)
But point of intersection of regression lines
$= (\bar{X}, \bar{Y})$
$\therefore \bar{X} = 13, \bar{Y} = 17$

41.(b) Given $P(A \cap B) = \frac{1}{4}$ $P(\bar{B}) = \frac{5}{8}$, So $P(B) = 1 - P(\bar{B})$
$= 1 - \frac{5}{8} = \frac{3}{8}$
$\therefore P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{3}{8}} = \frac{1}{4} \times \frac{8}{3} = \frac{2}{3}$

42.(c) We have, $\sim(p \Rightarrow q) \equiv p \wedge (\sim q)$
So the negation of "If battery is low then mobile doesnot work well" is battery is low and mobile works well.

43.(c) To have unique solution of AX = B, A^{-1} must exists and for this we must have $|A| \neq 0$

44.(d) Here, $\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$. So by definition the discontinuity is jump.

45.(a) $\lim_{n \to \infty} \frac{1 + 2 + 3 + ... + n}{n^2}$
$= \lim_{n \to \infty} \frac{n(n + 1)}{2n^2}$
$= \lim_{n \to \infty} \left(\frac{1}{2} + \frac{1}{n}\right) = \frac{1}{2} + 0 = \frac{1}{2}$

46.(c) Here, $\log_5 x = \log_5 e \log_e x$
So, $\frac{d\log_5 x}{dx} = \log_5 e \frac{d\log_e x}{dx} = \frac{\log_e 5}{x}$

47.(a) Suppose, $y = x^2$ So, $dy = 2x\, dx$
$= 2 \times 2 \times 0.01 = 0.04$

48.(c) Given $\frac{dy}{dx} = ye^x$ or, $\frac{dy}{y} = e^x dx$
Integrating $\int \frac{dy}{y} = \int e^x dx$
$\ln y = e^x + c$
Using y(0) = e, we have $\ln e = e^0 + c \Rightarrow c = 0$
Then $\ln y = e^x$ So $\ln y = e^1$ $\therefore y = e^e$

49.(b)	**50.(b)**	**51.(d)**	**52.(d)**	**53.(c)**	**54.(c)**
55.(d)	**56.(d)**	**57.(b)**	**58.(b)**	**59.(c)**	**60.(a)**

Section – II

61.(c) PE of dart = Energy of spring
or, $\frac{mgh_2}{mgh_1} = \left(\frac{x_2}{x_1}\right)^2 = \left(\frac{6}{4}\right)^2 = \frac{9}{4}$
$\therefore h_2 = \frac{9}{4} \times 2 = 4.5$ m

62.(a) $0 = \omega_0 + \alpha t$
or, $\alpha = -\frac{\omega_0}{t} = -\frac{2\pi f}{10} = -\frac{2\pi \times 20}{10} = -4\pi$ rad/s^2
$\tau = I\alpha$
$= 5 \times 10^{-3} \times 4\pi = \frac{\pi}{50}$ N

63.(a) Vol/s = Av
or, $70 \times 10^{-6} = 10^{-4}$ v
or, $70 \times 10^{-2} = \sqrt{2gh}$
or, $h = \frac{(70 \times 10^{-2})^2}{2g} = \frac{(70 \times 10^{-2})^2}{20}$
= 0.0245 m
= 2.45 cm

64.(d) $T_2 V_2^{\gamma-1} = T_1 V_1^{\gamma-1}$
or, $T_2 = 291 \left(\frac{V}{\frac{V}{8}}\right)^{1.4 - 1}$
$= 291(8)^{0.4} = 668$K
= 395°C
$\therefore \Delta T = 395 - 18 = 377$°C

65.(b) $\frac{\text{gain in time in 1 day}}{\text{lose in time in 1 day}} = \frac{\frac{1}{2}\alpha(\theta - 16) \times 1 \text{ day}}{\frac{1}{2}\alpha(40 - \theta) \times 1 \text{ day}}$
or, $\frac{5}{15} = \frac{(\theta - 16)}{(40 - \theta)}$
or, $3\theta - 48 = 40 - \theta$ or, $\theta = \frac{88}{4} = 22$°C

66.(c) $\frac{v}{4l_{c_1}} - \frac{v}{4l_{c_2}} = 4$
or, $\frac{v}{4}\left(\frac{1}{l_{c_1}} - 1\right) = 4$
or, $\frac{1}{l_{c_1}} = \frac{4 \times 4}{330} + 1 = \frac{346}{330}$
or, $l_{c_1} = \frac{330}{346} = 0.95$ m = 95 cm

67.(c) $\mu = \frac{\text{Real depth}}{\text{Apparent depth}}$
$\frac{4}{3} = \frac{x}{21 - x}$
$84 - 4x = 3x$
or, $x = \frac{84}{7} = 12$ cm

68.(a) $d\sin\theta_1 = \lambda$
or, $\sin\theta_1 = \frac{\lambda}{d}$
or, $\theta_1 = \sin^{-1}\left(\frac{\lambda}{d}\right) = \sin^{-1}\left(\frac{6328 \times 10^{-10}}{6.2 \times 10^{-3}}\right) = 0.18°$
$\therefore 2\theta_1 = 2 \times 0.18° = 0.36°$

69.(b) $Q = (C_1 + C) V'$
or, $C_1 V_1 = (C_1 + C) V'$

or, $C_1 + C = \frac{5 \times 10^{-6} \times 12}{3}$

or, $C = 20\ \mu F - 5\mu F = 15\ \mu F$

70.(c) Deflection fall 50 div to 10 div means $I = 5I_g$

$\therefore\quad S = \frac{I_g G}{I - I_g} \qquad G = \frac{12(5I_g - I_g)}{I_g} = 48\Omega$

71.(d) $E = \frac{d\phi}{dt} = AN\frac{dB}{dt}$

$= (0.1)^2 \times 500 \times 1$

$= 5V$

72.(b) **1st case**

$\frac{hc}{\lambda} = \phi + e \times 2.5V$

or, $ev = \left(\frac{hc}{\lambda} - \phi\right) \times \frac{1}{2.5}$ .... (1)

2nd case

$\frac{hc}{1.5\lambda} = \phi + ev$

or, $ev = \left(\frac{hc}{1.5\lambda} - \phi\right)$ ....(2)

From (1) & (2)

$\left(\frac{hc}{\lambda} - \phi\right)\frac{1}{2.5} = \frac{hc}{1.5\lambda} - \phi$

or, $\frac{hc}{2.5\lambda} - \frac{\phi}{2.5} = \frac{hc}{1.5\lambda} - \phi$

or, $\frac{hc}{1.5\lambda} - \frac{hc}{2.5\lambda} = \phi - \frac{\phi}{2.5}$

or, $\frac{hc}{\lambda}\left(\frac{1}{1.5} - \frac{1}{2.5}\right) = \frac{hc}{\lambda_0}\left(1 - \frac{1}{2.5}\right)$

or, $\lambda_0 = \frac{1.5}{2.5} \times \frac{2.5 \times 1.5}{1}\lambda$

$= 2.25\lambda$

73.(c) Initially

$N_0 = \frac{6.023 \times 10^{23}}{99} \times 10^{-12}$

$= 6.08 \times 10^9$

After 1 hr

$N = N_0\left(\frac{1}{2}\right)^{t/T_{1/2}}$

$= 6.08 \times 10^9\left(\frac{1}{2}\right)^{1/6} = 5.41 \times 10^9$

$A = \lambda N = \frac{0.693}{T_{1/2}} \times N$

$= \frac{0.693}{6} \times 5.41 \times 10^9 = 6.24 \times 10^8$ dis/hr

74.(b) Molar mass of $MgSO_4$ = 24 + 32 + (16×4) = 120 g/mol Moles = 9.6/120 = 0.08 mol

75.(c) Rate ∝ $[N_2][H_2]^3$. New rate = $(2)[N_2] \times (3[H_2])^3 = 2 \times 27 = 54$ times

76.(b) 2(+1) + 2x + 7(-2) = 0, where x is oxidation state of Cr 2 + 2x - 14 = 0, therefore x = +6

77.(d) As positive charge increases, ionic radius decreases (a is correct). Down the group, ionic radius increases (c is correct).

78.(a) Mn changes from +7 to +2, so electrons transferred = 5 Equivalent weight = Molecular weight/5 = 158/5

79.(c) In ethyne, each carbon forms 2 sigma bonds and has 2 pi bonds, indicating sp hybridization.

80.(b) Lucas reagent gives immediate turbidity with tertiary alcohols, turbidity after heating with secondary alcohols, and no reaction with primary alcohols.

81.(b) C_4H_{10} can exist as n-butane ($CH_3 - CH_2 - CH_2 - CH_3$) and isobutane ($CH_3 - CH(CH_3) - CH_3$)

82.(b) $-1 \le \sin3x \le 1$

So, $1 \le 2 - \sin2x \le 3$

$\Rightarrow\quad \frac{1}{3} \le \frac{1}{2\text{-}\sin3x} \le 1$

So range of $\left[\frac{1}{3}, 1\right]$

83.(c) $|x + iy - 1| = |x + iy - 1|$

$\Rightarrow\quad (x - 1)^2 + y^2 = x^2 + (y+1)^2$

$\Rightarrow\quad y = x$, st. line

84.(a) $\frac{(x\text{-}1)^n\,(1 - x)^n}{x^n} = \frac{(-1)^n(1\text{-}x)^{2n}}{x^n}$

Middle term = $T_{n+1} = \frac{(-1)^n}{x^n}\,2^n{}_{Cn}(-1)^n x^n = {}^{2n}C_n$

85.(b) Total number of ways in which 3 integers can be choosen is $^{20}C_3$.

The product is even if at least one of them is even.

Prob = 1 – prob of none of them is even

$= 1 - \frac{^{10}C_3}{^{20}C_3} = 1 - \frac{2}{19} = \frac{17}{19}$

86.(c) $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$

$\Rightarrow\quad (abc + 1 + 1) - (b + c + a) = 0$

$\Rightarrow\quad a + b + c = abc + 2$

87.(a) $bhx^2 + ahy^2 + 2abxy = 0$

$m_1 + m_2 = -\frac{2b}{h}$ and $m_1m_2 = \frac{b}{a}$

$m_2 = 2m_1$

$\Rightarrow\quad 3m_1 = -\frac{2b}{h}, 2m^2 = \frac{b}{a}$

$\Rightarrow\quad 2 \times \frac{4b^2}{9h^2} = \frac{b}{a} \quad\Rightarrow\quad ab: h^2 = 9 : 8$

88.(c) $y = -2x-\lambda$

$\therefore\quad -\lambda = -2(-2)(-2) + 2(-2)^3$

$\lambda = 24$

89.(b) abscissa $= \pm ae = \pm\sqrt{a^2 + b^2} = \pm\sqrt{\cos^2\alpha + \sin^2\alpha} = \pm 1$, no change

90.(c) $2xy\ dy = y^2dx - x^2dx$

$\frac{x\ dy^2 - y^2dx}{x^2} = -dx$

$d\left(\frac{y^2}{x}\right) = -dx$

$\frac{y^2}{x} = -x + c$

$y^2 = -x^2 + cx$

$x^2 + y^2 - cx = 0$

91.(a) $\lim_{x \to \infty} \left(\frac{x + a}{x + b}\right)^{nx + c}$

$= e^{n.(a - b)}$

So, $\lim_{x \to \infty} \left(\frac{x + 2}{x + 1}\right)^{x+3} = e^{1.(2-1)}$

$= e'$

$= e$

92.(a) $\int(x^3 + \frac{1}{e}e^x)dx = \frac{x^4}{4} + \frac{1}{e}e^x + c = \frac{x^4}{4} + e^{x-1} + c$

93.(c) $A = \frac{\sqrt{3}}{4}a^2 \Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}}{2}a.\frac{da}{dt}$

$\Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}}{2}ak$

94.(c) $\int_1^2 \ln x\ dx = [x\ln x - x]_1^2 = \ln\left(\frac{4}{e}\right)$

95.(c) $\left(\frac{a\sin C}{c}\right).\left(\frac{b\sin C}{c}\right) = \frac{ab}{c^2}$

$\sin^2 C = 1 \Rightarrow C = 90°$

96.(b) $\sin^{-1}\left(\frac{x}{5}\right) = \frac{\pi}{2} - \operatorname{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2} - \sin^{-1}\left(\frac{4}{5}\right)$

$\sin^{-1}\left(\frac{x}{5}\right) = \cos^{-1}\left(\frac{4}{5}\right) = \sin^{-1}\left(\frac{3}{5}\right)$

$\therefore\ x = 3$

97.(c) **98.(b)** **99.(b)** **100.(c)**

…The End…