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2083-04-06 Hints & Solution

Section – I

1.(d) Distance zero means particle is at same position i.e. displacement is zero.

2.(b) wt = mg, when body is taken from pole to equator, g decreases so wt decreases.

3.(d) $\Delta P = \frac{4T}{r}$

$r \propto \frac{1}{\Delta P}$

or, $\frac{r_1}{r_2} = \frac{(\Delta P)_{2nd}}{(\Delta P)_{1st}} = \frac{1}{3}$

$\frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$

4.(c) $\Delta F = (212 - 140)°F = 72°F$

$\frac{\Delta C}{\Delta F} = \frac{5}{9}$

or, $\Delta C = \frac{5}{g} \times 72 = 40°C$

5.(a) $\frac{1.004P}{P} = \frac{T+1}{T}$

or, $0.004T = T + 1$

or, $T = \frac{1}{0.004} = 250$ K

6.(b) $v = \frac{v_{max}}{2}$

or, $\omega\sqrt{A^2 - y^2} = \frac{1}{2}A\omega$

or, $A^2 - y^2 = \frac{A^2}{4}$

or, $y^2 = \frac{3A^2}{4}$

or, $y = \frac{\sqrt{3}}{2}A$

7.(b) Electric field intensity inside hollow sphere is zero.

8.(b) For maximum power

$R = r = 2\Omega$

$P = I^2R = \left(\frac{2E}{R+2}\right)^2 \times 2$

$= \left(\frac{2 \times 2}{2+2}\right)^2 \times 2 = 2W$

9.(c) $M = \sqrt{M_0^2 + 2M_0^2\cos 60 + M_0^2}$

$= \sqrt{3}\,M_0$

10.(a) Watt less mean P = 0

To be 0, ϕ must be 90°

11.(a) $0 = \sqrt{A_1A_2}$

12.(a) $\beta = \frac{D\lambda}{d}$

λ will be least for violet so β will be least for it.

13.(b) If uv doesnot cause photoelectric emission then it caused by photon of energy greater than uv i.e. x-ray.

14.(a) $V_{in} = I_bR_{in}$

or, $I_b = \frac{0.01}{1000} = 10^{-5}A$

$\beta = \frac{I_c}{I_b}$

or, $I_c = 50 \times 10^{-5} = 500\ \mu A$

15.(b) To find the mass of one molecule, divide the molar mass by Avogadro's number. The molar mass of H_2O is 18 g/mol.

16.(b) Each alkali metal has a characteristic flame color. Sodium is well-known for producing a bright golden yellow flame that is often used in street lamps.

17.(c) $FeCl_3$ is a salt of a weak base $Fe(OH)_3$ and strong acid HCl. The Fe^{3+} ion undergoes hydrolysis and acts as a Lewis acid in solution.

18.(d) Ionic radius decreases with increasing positive charge and across a period. Consider both the nuclear charge and the charge on the ion.

19.(b) Linkage isomerism occurs when a ligand can coordinate through different atoms. Look for ligands that have multiple potential donor atoms.

20.(c) Consider both the polarity of individual bonds and the overall molecular geometry. Even if bonds are polar, symmetrical geometry can result in zero net dipole.

21.(d) Paramagnetism increases with the number of unpaired electrons. Consider the electronic configurations and apply Hund's rule for maximum unpaired electrons.

22.(b) Electron affinity generally decreases down a group, but fluorine is an exception due to its very small size causing electron-electron repulsion.

23.(d) Isoelectronic species have the same number of electrons. Count the total electrons for each species after considering their charges.

24.(a) The classification depends on which subshell is being filled last. Look at the outermost electrons and the subshell that contains the highest energy electrons.

25.(a) The weaker the acid, the stronger its conjugate base. Consider the strength of the corresponding acids: HCl, H_2SO_4, CH_3COOH, and HF.

26.(c) Find the oxidation state of Mn in both species. MnO_4^- has Mn in +7 state, and Mn^{2+} has Mn in +2 state. Calculate the difference.

27.(c) In aqueous solution, consider the reduction potentials of Na^+ and H^+. The species with higher reduction potential gets reduced at the cathode.

28.(b) Aromatic compounds have specific structural requirements including planarity, conjugation, and follow Huckel's rule for stability.

29.(d)

30.(b) $\tan\theta = \frac{y}{x} = \frac{1 - \cos\frac{\pi}{5}}{\sin\frac{\pi}{5}} = \frac{2\sin^2\frac{\pi}{10}}{2\sin\frac{\pi}{10}\cos\frac{\pi}{10}} = \tan\frac{\pi}{10}$

$\therefore \quad \theta = \frac{\pi}{10}$

31.(a) $\alpha + \beta = -\frac{b}{a} = \frac{2}{3}$ and $\alpha\beta = \frac{c}{a} = -\frac{1}{3}$

Now, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{\frac{2}{3}}{-\frac{1}{3}} = -2$

32.(d) We have, $|\text{adj. } A| = |A|^{n-1}$, where n is the order of matrix A.

$\therefore \quad |\text{adj. } A| = |A|^{3-1} = 8^2 = 64$

33.(c) $\frac{2}{1!} + \frac{4}{3!} + \frac{6}{5!} + \ldots$

$= \frac{1+1}{1!} + \frac{3+1}{3!} + \frac{5+1}{5!} + \ldots$

$= 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \ldots$

34.(a) $\cos^{-1}(-x) - \sin^{-1}x$

$= \pi - \cos^{-1}x - \sin^{-1}x = \pi - (\cos^{-1}x + \sin^{-1}x)$

$= \pi - \frac{\pi}{2} = \frac{\pi}{2}$

35.(a) $s = \frac{a+b+c}{2} = 8$

$\sin\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}} = \sqrt{\frac{(8-4)(8-5)}{4 \times 5}} = \sqrt{\frac{3}{5}}$

36.(d) $\overrightarrow{AC} = 3\overrightarrow{AB}$

or, $\vec{c} - \vec{a} = 3(\vec{b} - \vec{a})$

$\Rightarrow \quad \vec{c} = 3\vec{b} - 2\vec{a}$

37.(c) We have $|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2 - (\vec{a}.\vec{b})^2$

or, $4^2 = (2^2)(5^2) - (\vec{a}.\vec{b})^2$

or, $(\vec{a}.\vec{b})^2 = 100 - 16$

or, $\vec{a}.\vec{b} = \sqrt{84} = 2\sqrt{21}$

38.(c)

39.(a) Since the triangle is right angled at (0, 0), the orthocentre is (0, 0)

40.(b) $a = 4, h = \frac{k}{2}, b = 1$

For coincident lines, $h^2 = ab$

or, $\frac{k^2}{4} = 4 \times 1$

or, $k^2 = 16 \Rightarrow k = \pm 4$

41.(b) $\frac{2b^2}{a} = b \Rightarrow \frac{b}{a} = \frac{1}{2}$

$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$

42.(d) Given planes are

$2x + y + 2z - 8 = 0$

i.e. $4x + 2y + 4z - 16 = 0$ ...(i)

and $4x + 2y + 4z + 5 = 0$ ... (ii)

$d = \left|\frac{d_1 - d_2}{\sqrt{a^2 + b^2 + c^2}}\right| = \left|\frac{-16 - 5}{\sqrt{4^2 + 2^2 + 4^2}}\right| = \frac{7}{2}$

43.(a) $\lim_{x \to \infty} \frac{\sin x}{x}$

$= \lim_{y \to 0} \frac{\sin\left(\frac{1}{y}\right)}{\frac{1}{y}} \quad \left[\text{Put } y = \frac{1}{x}\right]$

$= \lim_{y \to 0} y \sin\frac{1}{y} = 0$

44.(d) $\lim_{x \to 0} f(x) = f(0)$

or, $\lim_{x \to 0} \frac{1 - \cos x}{x^2} = k$

or, $\lim_{x \to 0} \frac{2\sin^2\frac{x}{2}}{x^2} = k$

or, $\lim_{x \to 0} 2\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}.2}\right)^2 = k$

or, $2 \times \frac{1}{4} = k$

$\therefore \quad k = \frac{1}{2}$

45.(d) $\frac{d\left(\sin^{-1}\frac{2x}{1+x^2}\right)}{d\left(\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right)} = \frac{d(2\tan^{-1}x)}{d(2\tan^{-1}x)} = 1$

46.(b) Here, $y = \int_0^x \frac{dt}{1+t^3}$

$\frac{dy}{dx} = \frac{1}{1+x^3}$

At $x = 1, \frac{dy}{dx} = \frac{1}{1+1} = \frac{1}{2}$

47.(c) $\mathbf{A} = \int_0^{\pi/4} y\,dx = \int_0^4 \tan x\, dx$

$= [l\text{nsec}x]_0^{\pi/4}$

$= l\text{nsec}\frac{\pi}{4} = l\text{n}\sqrt{2}$

$= \frac{1}{2} l\text{n}2$

48.(a) The eq^n can be written as

$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^9 = \left(\frac{d^2y}{dx^2}\right)^4$

$\therefore \quad$ Degree = 4

49.(a)	**50.(a)**	**51.(a)**	**52.(b)**	**53.(d)**	**54.(b)**
55.(b)	**56.(b)**	**57.(a)**	**58.(b)**	**59.(c)**	**60.(a)**

Section – II

61.(d) First case

$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 50} = \sqrt{1000}$

$= 31.6$ m/s

$v^2 = u^2 - 2gh'$

or, $h' = \dfrac{(31.6)^2 - (3^2)}{2 \times 2}$

$= 247.4$ m

$\therefore$ Height $= 50 + 247.4$

$= 297.4$ m

62.(a) $mg\sin\alpha = \mu mg\cos\alpha$

or, $\tan\alpha = \mu$

or, $\dfrac{1}{\cot\alpha} = \dfrac{1}{3}$

or, $\cot\alpha = 3$

63.(d) $\Delta PE = -\dfrac{GMm}{(R+h)} - \left(\dfrac{-GMm}{R}\right)$

$= GMm\left[\dfrac{1}{R} - \dfrac{1}{R+nR}\right]$

$= gR^2m\dfrac{(n+1-1)}{R(n+1)}$

$= mgR\,\dfrac{n}{n+1}$

64.(b) Heat lost by hot water = Heat gained by water + beaker.

or, $440(92 - \theta) = (200 + 20)(\theta - 20)$

or, $2(92 - \theta) = \theta - 20$

or, $184 - 2\theta = \theta - 20$

or, $204 = 3\theta$

or, $\theta = \dfrac{204}{3} = 68°C$

65.(b) $\left(\dfrac{T_2}{T_1}\right)^{\gamma} = \left(\dfrac{P_2}{P_1}\right)^{\gamma-1}$

or, $\dfrac{T_2}{300} = \left(\dfrac{A}{4A}\right)^{\frac{1.4-1}{1.4}}$

$T_2 = 201.8K$

$\Delta T = T_1 - T_2 = 300 - 201.8$

$= 98°C$

66.(a) $\dfrac{f_0'}{f_0} = \sqrt{\dfrac{T-U}{T}} = \sqrt{\dfrac{V\rho g - \dfrac{V}{2}.\sigma g}{V\rho g}}$

$= \sqrt{\dfrac{2\rho - \sigma}{\rho}}$

For water $\sigma = 1$ so $f_0' = 300\sqrt{\dfrac{2\rho - 1}{\rho}}$

67.(c) $C = C'$

or, $\dfrac{\varepsilon_0 A}{d} = \dfrac{\varepsilon_0 A}{d - t\left(1 - \dfrac{1}{\varepsilon_r}\right) + 2.4}$

or, $d = d - 3\left(1 - \dfrac{1}{\varepsilon_r}\right) + 2.4$

or, $2.4 = 3\left(1 - \dfrac{1}{\varepsilon_r}\right)$

or, $0.8 = 1 - \dfrac{1}{\varepsilon r}$

or, $\dfrac{1}{\varepsilon_r} = 1 - 0.8 = 0.2$

or, $\varepsilon_r = \dfrac{1}{0.2} = 5$

68.(d) $I^2R = E \times 2\pi r l$

or, $I^2\dfrac{\rho l}{\pi r^2} = E \times 2\pi r l$

$\therefore$ $I^2 \propto r^3$

$\left(\dfrac{r_2}{r_1}\right)^3 = \left(\dfrac{I_2}{I_1}\right)^2$

or, $\dfrac{r_2}{1} = \left(\dfrac{4}{0.5}\right)^{2/3} = 4$

$\therefore$ $r_2 = 4$mm

69.(d) $I = \dfrac{I_0}{2} = I_0(1 - e^{-Rt/L})$

or, $\dfrac{1}{2} = 1 - e^{-Rt/L}$

or, $e^{-Rt/L} = \dfrac{1}{2}$

or, $2 = e^{Rt/L}$

or, $ln2 = \dfrac{Rt}{L}$

or, $t = \dfrac{L}{R}\,ln2 = 1.4$ sec

70.(c) $\theta = \dfrac{\lambda}{d}$

or, $2\theta = \dfrac{2\lambda}{d}$

$\therefore$ $\theta_1 = \dfrac{2\lambda}{d}$

$\therefore$ $\dfrac{\theta_2}{\theta_1} = \dfrac{\lambda_2}{\lambda_1}$

or, $\lambda_2 = \dfrac{0.7\theta}{\theta} \times \lambda$

$= 0.7 \times 6000Å = 4200Å$

71.(a) $\tan C = \dfrac{r}{h}$

or, $h = \dfrac{\sin C}{\cos C}$

$= \dfrac{h}{\mu} \times \dfrac{1}{\sqrt{1 - \sin^2 C}}$

$= \dfrac{h}{\mu} \times \dfrac{\mu}{\sqrt{\mu^2 - 1}}$

$= \dfrac{12}{\sqrt{\left(\dfrac{4}{3}\right)^2 - 1}} = \dfrac{12 \times 3}{\sqrt{7}} = \dfrac{36}{\sqrt{7}}$ cm

72.(d) $\Delta V = \dfrac{hc}{e}\left(\dfrac{1}{\lambda_2} - \dfrac{1}{\lambda_1}\right)$

or, $V_2 - V_1 = \dfrac{hc}{e}\left(\dfrac{1}{3000 \times 10^{-10}} - \dfrac{1}{4000 \times 10^{-10}}\right)$

or, $V_2 = 2 + \dfrac{6.62 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19}}$

$\left(\dfrac{1}{3000 \times 10^{-10}} - \dfrac{1}{4000 \times 10^{-10}}\right)$

$= 2 + 1.03 = 3.03V$

73.(b) $\left(\frac{2}{3}\right)^{rd}$ deay means

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{t_1}{T_{1/2}}}$$

or, $\frac{1}{3} = \left(\frac{1}{2}\right)^{\frac{t_1}{T_{1/2}}}$

or, $t_1 = T_{1/2}\frac{ln\left(\frac{1}{3}\right)}{ln\left(\frac{1}{2}\right)} = 31.6$ min

$\left(\frac{1}{3}\right)^{rd}$ decay means

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{t_2}{T_{1/2}}}$$

or, $\frac{2}{3} = \left(\frac{1}{2}\right)^{\frac{t_2}{T_{1/2}}}$

or, $t_2 = T_{1/2}\frac{ln\left(\frac{2}{3}\right)}{ln\left(\frac{1}{2}\right)}$

$= 11.6$ min

$\Delta t = t_1 - t_2$

$= 31.6 - 11.6 = 20$ min

74.(b) Chlorine has 7 valence electrons. In ClF_3, three electrons are used for bonding with fluorine atoms. Calculate how many electrons remain and determine the number of lone pairs.

75.(c) For shell n = 4, you have 4s (1 orbital), 4p (3 orbitals), 4d (5 orbitals), and 4f (7 orbitals). Add them all up.

76.(c) Look at the oxidation states of the elements involved. Chlorine and iodine are both halogens - check if their oxidation states change during the reaction.

77.(c) Lewis acids are electron pair acceptors. Look for species that have incomplete octets or can expand their valence shell to accept electron pairs.

78.(b) Bond length is inversely related to bond order. CO has a triple bond, CO_2 has double bonds, and CO_3^{2-} has resonance structures with partial double bond character.

79.(a) This reaction requires heat input to break the chemical bonds in calcium carbonate. Consider whether energy is needed to drive the reaction forward.

80.(a) Nucleophilic addition rate depends on the electrophilicity of the carbonyl carbon. Consider both electronic and steric factors affecting the reactivity.

81.(c) In diamond, each carbon atom is bonded to four other carbon atoms in a tetrahedral arrangement. This geometry corresponds to a specific hybridization state.

82.(c) The function is defined when

$\log(x^2 - 6x + 6) \geq 0$

$\Rightarrow$ $x^2 - 6x + 6 \geq 1$

$\Rightarrow$ $(x - 5)(x - 1) \geq 0$

$\Rightarrow$ $(x \in (-\infty, 1] \cup [5, \infty($

83.(b)

First five questions	Remaining questions (13 – 5 = 8)	Selection
4	6	${}^5C_4 \times {}^8C_6 = 140$
5	5	${}^5C_5 \times {}^8C_5 = 56$

Required no. of choices = 140 + 56 = 196

84.(b) $(x + a)^{100} + (x - a)^{100}$

$= 2(x^{100} + {}^{100}C_2\, x^{98}a^2 + {}^{100}C_4 x^{96}a^4 + ... + {}^{100}C_{100}\, a^{100})$

Number of terms = 51

85.(c) Let no. of sides = n

Sum of interior angles of a polygon = $(n - 2)\pi$

The angles are in AP

$a = 120°, d = 5°$

Then, $\frac{n}{2}[2 \times 120 + (n - 1)5] = (n - 2)\, 180°$

or, $n^2 - 25n + 144 = 0$

$n = 9, 16$

But n = 16 is not possible as

$t_{16} = a + 15d = 120 + 15 \times 5 = 195$ which is greater than 180°

86.(c) $\tan(\cot x) = \cot(\tan x)$

$= \tan\left(\frac{\pi}{2} - \tan x\right)$

or, $\cot x = n\pi + \frac{\pi}{2} - \tan x$

or, $\cot x + \tan x = n\pi + \frac{\pi}{2}$

or, $\frac{2}{\sin 2x} = n\pi + \frac{\pi}{2}$

$\therefore$ $\sin 2x = \frac{4}{(2n + 1)\pi}$

87.(b) $|\vec{a}.\vec{b}| = ab\cos\theta = 3$ ....(i)

$|\vec{a} \times \vec{b}| = ab\sin\theta = 4$ .... (ii)

Dividing (ii) by (i)

$\tan\theta = \frac{4}{3}$

$\Rightarrow$ $\cos\theta = \frac{3}{5} \Rightarrow \theta = \cos^{-1}\left(\frac{3}{5}\right)$

88.(a) $\frac{b_{xy} + b_{yx}}{2} = p \Rightarrow b_{xy} + b_{yx} = 2p$ ...(i)

and $b_{xy} - b_{yx} = 2q$ ... (ii)

Then, $b_{xy} = p + q$

and $b_{yx} = p - q$

We have, $r = \sqrt{b_{xy}.b_{yx}} = \sqrt{(p + q)(p - q)}$

$= \sqrt{p^2 - q^2}$

89.(d) Given, circle is

$x^2 + y^2 - 2x = 0$ ...(i)

and line is $y = x$ .... (ii)

Put $y = x$ in (i), we get

$2x^2 - 2x = 0 \Rightarrow x = 0, 1$

From (i) $y = 0, 1$

Then, the coordinates of A and B are (0, 0) and (1, 1)

The required eq^n is

$(x - 0)(x - 1) + (y - 0)(y - 1) = 0$

$\therefore \quad x^2 + y^2 - x - y = 0$

90.(c) The parabola is

$$y^2 = 4.\frac{k}{4}\left(x - \frac{8}{k}\right)$$

The eq^n of directrix is

$$x - \frac{8}{k} + \frac{k}{4} = 0$$

But $x - 1 = 0$ is the directrix

So, $\frac{8}{k} - \frac{k}{4} = 1 \Rightarrow k = -8, 4$

91.(a) $a_1 = 2, b_1 = 2, c_1 = 1$

$a_2 = 7 - 3 = 4, b_2 = 2 - 1 = 1,$

$c_2 = 12 - 4 = 8$

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$= \frac{2 \times 4 + 2 \times 1 + 1 \times 8}{\sqrt{2^2 + 2^2 + 1^2}\sqrt{4^2 + 1^2 + 8^2}}$$

$$= \frac{2}{3}$$

$$\therefore \quad \theta = \cos^{-1}\left(\frac{2}{3}\right)$$

92.(c) $\lim\limits_{x \to 2} \frac{xf(2) - 2f(x)}{x - 2} \left[\frac{0}{0} \text{ form}\right]$

$= \lim\limits_{x \to 2} \frac{f(2) - 2f'(2)}{1}$

$= 4 - 2 \times 4 = -4$

93.(b) Let $x < 0$, $|x| = -x$

Then, $f'(x) = \frac{d}{dx}\left(\frac{x}{1 - x}\right) = \frac{1}{(1 - x)^2}$

At $x = 0$, $f'(0) = 1$

Again, let $x > 0$, $|x| = x$

Then, $f'(x) = \frac{d}{dx}\left(\frac{x}{1 + x}\right) = \frac{1}{(1 + x)^2}$

At $x = 0$, $f'(0) = 1$

$\therefore \quad f'(0) = 1$

94.(a) $y = x^x$

$ln y = x ln x$

$\frac{1}{y}\frac{dy}{dx} = x.\frac{1}{x} + ln x.1 = 1 + ln x$

$\frac{dy}{dx} = x^x(1 + ln x)$

For stationary point, $\frac{dy}{dx} = 0$

$\Rightarrow \quad 1 + ln x = 0$

$\Rightarrow \quad ln x = -1$

$\Rightarrow \quad x = e^{-1} = \frac{1}{e}$

95.(b) Put $t = x^2 \Rightarrow dt = 2xdx$

Now, $\int \frac{x}{1 + x^4} dx = \frac{1}{2}\int \frac{1}{1 + t^2} dt = \frac{1}{2}\tan^{-1}(t) + c$

$= \frac{1}{2}\tan^{-1}(x^2) + c$

96.(d) Area $= 2\int_0^1 y dx$

$= 2\int_0^1 (1 - |x|)dx$

$= 2\int_0^1 (1 - x)dx$

$= 2\left[x - \frac{x^2}{2}\right]_0^1$

$= 2\left(1 - \frac{1}{2}\right) - 0 = 1$

97.(b) **98.(c)** **99.(a)** **100.(d)**

...Best of Luck...