1.(d) For the positive side of the y-axis
(Where $y \geq 0$)
In the 1st quadrant; $y = x$
In the 2nd quadrant; $y = -x$
Gives the result $y = |x|$

2.(c) $\sum_{n=1}^{n} 2^n = 2 + 2^2 + 2^3 + ... + 2^n$

$$= \frac{2(2^n - 1)}{2 - 1} = 2.2^n - 2 = 2^{n+1} - 2$$

3.(c) $ax^2 + bx + c$

$\alpha + \beta = -\frac{b}{a}, \alpha\beta = \frac{c}{a}$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$$

$$= \frac{\left(-\frac{b}{a}\right)^2 - 2 \times \frac{c}{a}}{\left(\frac{c}{a}\right)^2} = \frac{b^2 - 2ac}{c^2}$$

4.(b) $\sin\theta = \frac{1}{\sqrt{2}}, \cos\theta = \frac{1}{\sqrt{2}}$

$\sin\theta = \sin\frac{\pi}{4}, \cos\theta = \cos\frac{\pi}{4}$

$\therefore \quad \theta = 2n\pi + \frac{\pi}{4}$

5.(a) $n(X) = 55, n(Y) = 50, n(X \cup Y) = 90$
$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$
$n(X \cap Y) = 55 + 50 - 90$
$\therefore \quad n(X \cap Y) = 15$

6.(b) **Check:**
$a^2 + b^2 = c^2 \Rightarrow$ Right angle triangle.

7.(a) $$\begin{vmatrix} -3 & 0 & 0 \\ 6 & 4 & 0 \\ -10 & 2 & 5 \end{vmatrix} = -3 \begin{vmatrix} 4 & 0 \\ 2 & 5 \end{vmatrix} - 0 + 0$$

$$= -3 \times (20 - 0) = -60$$

8.(b) $5x^2 + 13x + K = 0$

Let α and $\frac{1}{\alpha}$ be two roots then

$\alpha \times \frac{1}{\alpha} = \frac{c}{a} = \frac{K}{5}$

$\therefore \quad K = 5$

9.(b) $R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$= (a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

$$= (a+b+c) \begin{vmatrix} 0 & 0 & 1 \\ a+b+c & -(a+b+c) & 2b \\ 0 & a+b+c & c-a-b \end{vmatrix}$$

$= (a+b+c) \times (a+b+c)^2$
$= (a+b+c)^3$

10.(a) $(1 + \omega - \omega^2)^4 + (1 - \omega + \omega^2)^4$
$= (-2\omega^2)^4 + (-2\omega)^4$
$= 16\omega^8 + 16\omega^4$
$= 16(\omega^2 + \omega)$
$= -16$

11.(b) $I = \int \frac{e^x}{1 + e^x} dx$

Note: $\int \frac{f'(x)}{f(x)} dx = \log[f(x)] + c$

$\therefore \quad I = \log(1 + e^x) + c$

12.(c) $I = \int \left(x + \frac{1}{x}\right) dx$

$= \frac{x^2}{2} + \log x + c$

13.(a) $I = \int \sqrt{x^2 - 4}\, dx$
$= \int \sqrt{x^2 - 2^2}\, dx$
$= \frac{x\sqrt{x^2-4}}{2} - 2\log(x + \sqrt{x^2 - 4}) + c$

14.(d) $I = \int_0^{\frac{\sqrt{3}}{2}} \frac{dx}{\sqrt{1 - x^2}}$

$= [\sin^{-1}x]_0^{\sqrt{3}/2}$

$= \sin^{-1}\frac{\sqrt{3}}{2} - \sin^{-1}0 = \frac{\pi}{3} - 0 = \frac{\pi}{3}$

15.(d) $\hat{AB} = \frac{\vec{AB}}{|\vec{AB}|}$

$= \frac{(3-1)\vec{i} + (2-0)\vec{j} + (0-1)\vec{k}}{\sqrt{(3-1)^2 + (2-0)^2 + (0-1)^2}}$

$= \frac{2\vec{i} + 2\vec{j} - \vec{k}}{3}$

$= \frac{2}{3}\vec{i} + \frac{2}{3}\vec{j} - \frac{1}{3}\vec{k}$

16.(b) $l^2 + (-2l)^2 + (3l)^2 =$ $[\because l^2 + m^2 + n^2 = 1]$
$l^2 + 4l^2 + 9l^2 = 1$
$14l^2 = 1$
$l^2 = \frac{1}{14}$
$\therefore \quad l = \pm\frac{1}{\sqrt{14}}$

17.(d) Let $A = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$

$\therefore \quad A^{-1} = \frac{1}{\sqrt{|A|}}$ Adjoint matrix of A

$= \frac{1}{10}\begin{bmatrix} 4 & -1 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 0.4 & -0.1 \\ -0.2 & 0.3 \end{bmatrix}$

18.(a) $\sin\alpha + \cos\alpha = -\frac{b}{a}$, $\sin\alpha.\cos\alpha = \frac{c}{a}$

Now, $\sin^2\alpha + \cos^2\alpha = 1$

$(\sin\alpha + \cos\alpha)^2 - 2\sin\alpha.\cos\alpha = 1$

$\left(-\frac{b}{a}\right)^2 - 2 \times \frac{c}{a} = 1$

$b^2 - 2ac = a^2$

$a^2 + 2ac - b^2 = 0$

19.(c) $\cot^{-1}3 + \operatorname{cosec}^{-1}\sqrt{5}$

$\Rightarrow \quad \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{2}\right)$

$\Rightarrow \quad \tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{2}}{1-\frac{1}{3}\times\frac{1}{2}}\right) = \tan^{-1}(1) = \frac{\pi}{4}$

20.(b) $2x + y = 7$ and $x + 3y = 11$

Let $A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$

Adjoint matrix of $A = \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}$

21.(d) Eq^n of circle: $(x-0)^2 + (y-1)^2 = \sqrt{(0-0)^2 + (1-0)^2}$

$x^2 + y^2 - 2y + 1 = 1$

$\therefore \quad x^2 + y^2 - 2y = 0$

22.(d) $\frac{x^2}{25} + \frac{y^2}{16} = 1$

Eccentricity $(e) = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{16}{25}}$

$= \frac{3}{5}$

23.(b) $x = ty - at^2$, $y^2 = 4ax$

$\Rightarrow \quad y^2 = 4a(ty - at^2)$

$\Rightarrow \quad (y - 2at)^2 = 0$

$\therefore \quad y = 2at$, $x = t(2at) - at^2$

$x = at^2$

24.(a) L.R = 3

$\frac{2b^2}{a} = 3 \Rightarrow b^2 = \frac{3a}{2}$

$e = \sqrt{1 - \frac{b^2}{a^2}}$

$\Rightarrow \quad \frac{1}{2} = 1 - \frac{3a}{2a^2}$

$\Rightarrow \quad \frac{3}{2a} = \frac{1}{2}$

$\Rightarrow \quad a = 3, b^2 = \frac{9}{2}$

$\Rightarrow \quad \frac{x^2}{9} + \frac{y^2}{\frac{9}{2}} = 1 \Rightarrow x^2 + 2y^2 = 9$

25.(b) $a = 0, h = \frac{k}{2}, b = 0$

$g = -4, f = \frac{9}{2}, c = -12$

Use: $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$

Solving; we get

$k = 0, 6$ but 0 is not possible

26.(a) $ax^2 + 2hxy + by^2$ reprosents a pair of straight lines passing through origin and parallel to the original pair of lines.

27.(d) $\lim_{h \to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h} \left(\frac{0}{0} \text{ form}\right)$

$= \lim_{h \to 0} \frac{1}{2\sqrt{2+h}} = \frac{1}{2\sqrt{2}}$

28.(b) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

29.(d) $I = \int 1.\log x\, dx$

$I = \log x \int 1.dx - \int \left(\frac{d(\log x)}{dx} \int I.dx\right) dx$

$= x\log x - \int \frac{1}{x} \times x dx$

$= x\log x - x + c$

30.(c) $f(x) = \begin{cases} x\sin\frac{1}{x} & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$

$-1 \le \sin\left(\frac{1}{x}\right) \le 1$

$-|x| \le x\sin\left(\frac{1}{x}\right) \le |x|$

Since, $\lim_{x \to 0} -|x| = 0$ and $\lim_{x \to 0} .|x| = 0$

$\therefore \quad \lim_{x \to 0} x\sin\left(\frac{1}{x}\right) = 0$

31.(b) Sample space = {HHH, HHT, HTH, HTT, THH, HTT, THT, TTT}

$\therefore$ Probability of getting 2 tails

$= \frac{\text{No. of favourable outcome}}{\text{No. of total outcomes}} = \frac{3}{8}$

32.(a) $\bar{x} = \frac{a + a + a + a}{4} = a$

But $x - \bar{x} = a - a = 0$ for all

$\therefore$ Standard deviation = 0

$$\left[\because \sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}\right]$$

33.(d) If $2\vec{i} + \lambda\vec{j} + \vec{k}$ and $\vec{i} - 2\vec{j} + 4\vec{k}$ are perpendicular then

$\Rightarrow \quad 2 \times 1 + (\lambda \times -2) + (1 \times 4) = 0$

$\Rightarrow \quad 2 - 2\lambda + 4 = 0$

$\Rightarrow \quad 2\lambda = 6$

$\Rightarrow \quad \lambda = 3$

34.(d) $(2 + x)^{-n}$

$2^{-n}\left(1 + \frac{x}{2}\right)^{-n}$

Validity: $\left(\frac{x}{2}\right) < 1$

$-1 < \frac{x}{2} < 1$

$-2 < x < 2 \Rightarrow |x| < 2$

35.(b) $f(x) = \frac{1}{x}$

$f'(x) = -\frac{1}{x^2}$; except $x = 0$, $x^2 > 0$ so, the $f'(x) = -\frac{1}{x^2}$ always negative.

Hence continuous decreasing on $(-\infty, 0)$ and $(0, \infty)$

36.(d) $y\sin x = x + c$

D.w.r to x

$\sin x\frac{dy}{dx} + y\cos x = 1$

$\frac{dy}{dx} + \cot x.y = \operatorname{cosec} x$

$\therefore \quad P = \cot x, Q = \operatorname{cosec} x$

37.(c) $(1 + 3x)^2 \times (1 - 2x)^{-1}$

$\Rightarrow \quad (1 + 6x + 9x^2) \times (1 + 2x + 4x^2 + 8x^3 + 16x^4 + ...)$

$\therefore \quad$ Coefficient of $x^4 = 1 \times 16 + 6 \times 8 + 9 \times 4 = 100$

38.(c) $\text{Area} = \int_0^{\pi/4} \tan x\,dx + \int_{\pi/4}^{\pi/2} \cot x\,dx$

$= [\log\sec x]_0^{\pi/4} + [\log\sin x]_{\pi/4}^{\pi/2}$

$= \log\sqrt{2} - \frac{1}{\log\sqrt{2}}$

$= \log 2$

39.(d) $f(x) = 1 - |x - 7|$

Let; $y = f(x)$

$y = 1 - |x - 7|$

$|x - 7| = 1 - y$

$|x - 7| \geq 0$

$1 - y \geq 0$

$y \leq 1$

Range $= (-\infty, 1]$

40.(b) $\frac{1}{\log_a x} + \frac{1}{\log_c x} = \frac{2}{\log_b x}$

$\log_x a + \log_x c = 2\log_x b$

$\log_x ac = \log_x b^2$

$\therefore \quad b^2 = ac \Rightarrow$ G.P

41.(c) $W = KE$

or, $Ees = \frac{1}{2}mv^2$

or, $v = \sqrt{\frac{2Ees}{m}}$

42.(a) Initially $R = \frac{\rho l}{A}$

For each part $R' = \frac{\rho l}{4A} = \frac{R}{4}$

In parallel $R_2 = \frac{R}{4} \times \frac{1}{4}$

$\therefore \quad \frac{R_2}{R} = \frac{1}{16}$

43.(c) $E_1 = 13.6\text{ eV} = 136 \times 1.6 \times 10^{-19}\text{ J}$

$= 2.17 \times 10^{-18}\text{ J}$

44.(b) $\frac{m}{m_0} = \left(\frac{1}{2}\right)^{t/T_{1/2}} = \left(\frac{1}{2}\right)^{100/25} = \frac{1}{16}$

$m = \frac{1}{16} \times 2 = \frac{1}{8}\text{ kg}$

Mass decayed $= m_0 - m = 2 - \frac{1}{8}$

$= \frac{15}{8}$

$= 1.875\text{ kg}$

45.(c) Vel. gradient $= \frac{dV}{dL} = \frac{LT^{-1}}{L} = [T^{-1}]$

46.(c) $a = r\omega^2 = 0.25\,(2\pi f)^2$

$= 4\pi^2 \times 0.25 \times 2^2$

$= 4\pi^2\text{ m/s}^2$

47.(c) $\text{Stress} = Y_A\alpha_A\Delta\theta = Y_B\alpha_B\Delta\theta$

or, $Y_A\alpha = Y_B 2\alpha$

or, $Y_A = 2Y_B$

48.(a) Change in SVP for 1°C = 0.6 mm of Hg

Change in SVP for 0.2°C = 0.12 mm of Hg

$\therefore \quad$ SVP at 14.2°C = 12.6 + 0.12

$= 12.72$ mm of Hg

49.(b) $\mu = \frac{\text{Real depth}}{\text{Apparent depth}}$

or, $\text{Apparent depth} = \frac{3.5}{1.309}$

$= 2.67\text{ cm}$

50.(a) $\frac{v_H}{v_O} = \sqrt{\frac{M_O}{M_H}} = \sqrt{\frac{32}{2}} = 4$

$v_H = 4v_0$

51.(b) Near point = 100 cm

u = 25 cm

v = –100 cm

$$f = \frac{uv}{u+v} = \frac{25(-100)}{25-100} = \frac{2500}{75} = \frac{100}{3}\text{ cm} = \frac{1}{3}\text{ m}$$

$$P = \frac{1}{f_{\text{in metre}}} = \frac{1}{\frac{1}{3}} = 3\text{D}$$

52.(d) $e = \frac{l_2 - 3l_1}{2}$

53.(c) While approaching

$$f' = \frac{v}{v - v_s} \times f$$

$$100 = \frac{v}{v - v_s} f \ldots (1)$$

When passes

$$f'' = \frac{v}{v + v_s} \times f$$

$$\text{or,} \quad 50 = \frac{v_f}{v + v_s} \ldots (2)$$

Dividing (1) by (2)

$$\frac{100}{50} = \frac{v + v_s}{v - v_s}$$

$$\text{or,} \quad 2v - 2v_s = v + v_s$$

$$\text{or,} \quad v_s = \frac{v}{3}$$

$$\text{From (1)} \quad 100 = \frac{v}{v - \frac{v}{3}} f = \frac{v}{\frac{2v}{3}} f$$

$$\text{or,} \quad f = \frac{200}{3} = 66.7\text{ Hz}$$

54.(c) $S = \frac{IgG}{I - Ig} = \frac{25 \times 10^{-3} \times 5}{10 - 25 \times 10^{-3}} = 0.012\ \Omega$

55.(a) $R \times t = CV$

$$\text{or,} \quad t = \frac{800 \times 10^{-6} \times 10}{40 \times 10^{-6}} = 200\text{ sec}$$

56.(b) $P = \frac{V^2}{R}$

$$R = \frac{100^2}{500} = 20\Omega$$

$P = IV$

$$I = \frac{500}{100} = 5\text{A}$$

$$\text{Now } I = \frac{200}{R + 20}$$

$$R + 20 = \frac{200}{5}$$

$$\text{or,} \quad R = 20\Omega$$

57.(c) $E = -L\frac{dI}{dt}$

$= -5(-2) = +10\text{V}$

58.(b) $v = \frac{C}{137} \times \frac{Z}{n}$

For Hydrogen Z = 1 so

$$v = \frac{C}{4 \times 137} = \frac{C}{548}$$

59.(c) $\frac{mv^2}{r} = R + mg$

$$R = \frac{mv^2}{r} - mg$$

$$= \frac{5 \times 4^2}{0.2} - 5 \times 10$$

$= 50 \times 8 - 50 = 350\text{ N}$

60.(d) $\lambda = \frac{h}{mv}$

$$\text{or,} \quad v = \frac{h}{m\lambda} = \frac{6.62 \times 10^{-34}}{1.67 \times 10^{-27} \times 0.2 \times 10^{-9}}$$

$= 1982\text{ m/s}$

61.(b) $eV = \frac{1}{2}mv^2$

$$\text{or,} \quad v = \sqrt{\frac{2eV}{m}} \qquad \text{i.e. } v \propto \sqrt{V}$$

$$\text{or,} \quad \frac{v'}{v} = \sqrt{\frac{V'}{V}}$$

$$\text{or,} \quad v' = 6 \times 10^6 \sqrt{\frac{8}{2}} = 12 \times 10^6\text{ m/s}$$

62.(b) $KE = eV_s$

$$\text{or,} \quad V_s = \frac{2eV}{e} = 2V$$

–ve potential of anode

63.(c) $v_R = \sqrt{v_x^2 + v_y^2}$

While moving up $v_y = \sin\theta - gt$

$$\therefore \quad v_R = \sqrt{(v\cos\theta)^2 + (v\sin\theta - gt)^2}$$

$$= \sqrt{v^2\cos^2\theta + v^2\sin^2\theta - 2vgt\sin\theta}$$

$$= \sqrt{v^2 - 2vgt\sin\theta + g^2t^2}$$

64.(a) $V_0 = A\omega$

$$V = \omega\sqrt{A^2 - y^2} = \omega\sqrt{A^2 - \left(\frac{A}{2}\right)^2}$$

$$= \frac{\sqrt{3}}{2} A\omega = \frac{\sqrt{3}}{2} V_0$$

65.(b) $KE_1 = KE_2$

$$\text{or,} \quad \frac{P_1^2}{4m_1} = \frac{P_2^2}{4m_2}$$

$$\text{or,} \quad \frac{P_1^2}{4 \times 4} = \frac{P_2^2}{4 \times 16}$$

$$\text{or,} \quad P_1 = \frac{P_2}{2}$$

66.(b) $PV = nRT$ ....(i)

$2P2V = nRT'$ .... (ii)

or, $\frac{T'}{T} = 4$

or, $T' = 4T$

67.(b) $60\%\ of\ mgh = \frac{m}{10} L_f$

or, $h = \frac{80 \times 4200}{10 \times 0.6 \times 10} = 5600\ m = 5.6\ km$

68.(b) $\frac{V}{v} = (n)^{2/3}$

$V = (27)^{2/3}\ V = 9V$

69.(c) For radio turning

$f_0 = \frac{1}{2\pi\sqrt{LC}}$

$= \frac{1}{2\pi\sqrt{5 \times 6 \times 10^{-6}}}$

$= 29.05\ Hz$

$\lambda = \frac{C}{f_0} = \frac{3 \times 10^8}{29.05} = 1.03 \times 10^7\ m$

70.(a) For central maxima

$2y = \frac{2D\lambda}{d}$

$2y = d,\ D = x$ so

$d = \frac{2\lambda x}{d}$

or, $x = \frac{d^2}{2\lambda}$

71.(a) For E_1 and E_2 reaction

Order of reactivity: 3° > 2° > 1° due to stability of alkene.

72.(b) $2KMnO_4 + 8HCl \rightarrow 2Ku + 2MnCl_2 + Cl_2 + 4H_2O$

$Cl_2 + 2NaOH \rightarrow NaCl + NaOCl + H_2O$

73.(c)

74.(d) $(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} N_2 + Cr_2O_3 + 4H_2O$

75.(b)

76.(c) 3-hydroxy-3-methyl-butan-2-one because ketone is main functional group.

77.(d)

78.(c) Benzene has delocalized π-electrons due to resonance.

79.(a) $2H_2 + O_2 \longrightarrow 2H_2O$

2mole 1mole 2mole

32gm 36gm

$1.5\ mole = \frac{32}{2} \times 1.5\ gm = 24\ gm$

We have only 20gm O_2 so it is limiting reagent.

Now, 32gm O_2 gives 36gm H_2O

20gm O_2 gives $\frac{36}{32} \times 20gm\ H_2O$

$= 22.5\ gm$

80.(c)

81.(b) $[H^+] = 5 \times 10^{-3}\ N$

$pH = -\log[H^+] = -\log(5 \times 10^{-3})$

$= 3 - \log 5 \approx 3 - 0.7 = 2.3$

82.(d)

83.(c)

84.(c)

85.(b) $K_2Cr_2O_7 + 3SO_2 + H_2SO_4 \rightarrow Cr_2(SO_4)_3 + K_2SO_4 + H_2O$

(Green)

86.(c) $2Na + Cl_2 \rightarrow 2NaCl$

87.(c)

88.(b)

89.(d) Mol. mass of $Na_2CO_3 = 106\ g/mol$

Equivalent of $Na_2CO_3 = \frac{0.265g}{106g/mol} \times 2$

$= 0.005$ equivalent

Equivalents of $H_2SO_4 = 0.1N \times 0.12$

$= 0.01$ equivalent

Remaining acid $= 0.01 - 0.005 = 0.005$ eq. of H^+

Now, $[H^+] = \frac{0.005\ eq.}{0.12} = 0.05m$

$pH = -\log(0.05) \approx 1.3$

90.(a) $CaCO_3 \rightarrow CaO + CO_2$

Moles of $CaCO_3 = \frac{1g}{100g/mol} = 0.01$ mol.

Volume of CO_2 at NTP $= 0.01\ mole \times 22400\ ml/mol$

$= 224\ ml$

91.(b) **92.(d)** **93.(d)** **94.(b)** **95.(a)**
96.(c) **97.(b)** **98.(c)** **99.(a)** **100.(b)**

...Best of Luck...