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2083-3-27 Hints & Solution

Section – I

1.(a) $\vec{P} \times \vec{Q} = |\vec{P}|\,|\vec{Q}| = \sin\hat{n} = 0$

$\therefore \quad \sin\theta = \sin 0°$

or, $\quad \theta = 0°$

$R = \sqrt{P^2 + 2PQ\cos 0° + Q^2} = \sqrt{(P+Q)^2} = P + Q$

2.(c) When body crosses top most position with critical velocity the velocity at horizontal position is $v = \sqrt{3gr}$

$a = \frac{v^2}{r} = \frac{3gr}{r} = 3g$

3.(b) $\frac{v_{e_1}}{v_{e_2}} = \frac{\sqrt{\frac{2GM_1}{R_1}}}{\sqrt{\frac{2GM_2}{R_2}}} = \sqrt{\frac{M_1}{R_1} \times \frac{R_2}{M_2}}$

$= \sqrt{\frac{\frac{4\pi}{3}\frac{R^2\rho}{R} \times 2R}{\frac{4\pi}{3}(2R)^3\rho}}$

$= \frac{R^2}{4R^2} = \frac{1}{2}$

4.(c) $T = \text{stress} \times A = mr\omega^2$

or, $\quad \omega = \sqrt{\frac{\text{Stress} \times A}{ml}}$

$= \sqrt{\frac{4.8 \times 10^7 \times 10^{-6}}{10 \times 0.3}} = 4 \text{ rad/s}$

5.(c) $\omega t_1 = \omega t_2$

Again $\omega t_1 - (\text{upthrust})_1 = \omega t_2 - (\text{upthrust})_2$

or, $\quad (\text{upthrust})_1 = (\text{upthrust})_2$

or, $\quad 2 \times \rho_0 g = V\rho_l g$

or, $\quad \rho_l = 2 \times 0.9 = 1.8 \text{ g/cc}$

6.(b) $\alpha = \frac{\Delta l}{l\Delta\theta}$

or, $\quad \Delta\theta = \frac{1.1 \times 10^{-3}}{10 \times 1.1 \times 10^{-5}} = 10°C$

7.(b) $VP^2 = \text{constant}$

or, $\quad V\left(\frac{nRT}{V}\right)^2 = \text{constant}$

or, $\quad \frac{T^2}{V} = \text{constant}$

or, $\quad \frac{T^2}{2V} = \frac{T_0^2}{V}$

or, $\quad T = \sqrt{2}\,T_0$

8.(d) $f_0^0 = 3f_0^c$

or, $\quad \frac{v}{2l_0} = 3 \times \frac{v}{4l_c}$

or, $\quad \frac{l_0}{l_c} = \frac{2}{3}$

9.(c) $\frac{R'}{R} = \left(\frac{d}{d'}\right)^4 = \left(\frac{d}{\frac{d}{3}}\right)^4 = 81$

$R' = 81R$

10.(a) $F = B_1 Il$

$= \frac{\mu_0 I}{2\pi a} Il$

$= \frac{4\pi \times 10^{-7} \times 1 \times 1 \times 0.5}{2\pi \times 1}$

$= 10^{-7}$ N

11.(a) $I = \frac{V_{rms}}{X_L} = \frac{V_0}{\sqrt{2} \times \omega L}$

$= \frac{282}{\sqrt{2} \times 314 \times 0.1}$

$= 6.35$ A

12.(b) $I = \frac{ne}{t}$

$\frac{n}{t} = \frac{I}{e} = \frac{4.8 \times 10^{-3}}{1.6 \times 10^{-19}}$

$= 3 \times 10^{16}$

13.(b) $\lambda = \frac{h}{mv} = \frac{6.62 \times 10^{-34}}{9.1 \times 10^{-31} \times 2 \times 10^6}$

$= 3.64 \times 10^{-10}$ m

$= 3.64$ Å

14.(c) $R = \frac{\Delta V}{\Delta I} = \frac{0.7 - 0.5}{10^{-3}} = 200\Omega$

15.(c) **Hint:** One molecule of CO_2 contains 3 atoms (1 C + 2 O). Multiply by Avogadro's number.

Solution: 1 molecule of CO_2 has 3 atoms.

Total atoms = $3 \times 6.022 \times 10^{23} = 1.806 \times 10^{24}$.

16.(b) **Hint:** Find the molar mass of NH_4NO_3, then take the mass of the two nitrogen atoms as a percentage.

Solution: Molar mass of NH_4NO_3 = 14 + 4 + 14 + 48 = 80.

Mass of N = 2 × 14 = 28.

%N = (28/80) × 100 = 35%.

17.(b) **Hint:** This is Gay-Lussac's (pressure) law. Pressure and temperature must both be measured on the absolute (Kelvin) scale.

Solution: By Gay-Lussac's law, at constant volume $P \propto T$, where T is the absolute (Kelvin) temperature.

18.(c) **Hint:** The number of orbitals in a shell equals n^2. (The maximum number of electrons would be $2n^2$.)

Solution: Number of orbitals = $n^2 = 4^2 = 16$. (These 16 orbitals can hold a maximum of $2n^2 = 32$ electrons.)

19.(d) **Hint:** All four belong to the same group (alkali metals). Atomic radius increases down a group.

Solution: Down group 1, atomic radius increases (more shells). Rb is lowest in the group, so it has the largest atomic radius.

20.(d) **Hint:** Take oxygen as −2 and remember the overall charge on the ion is −1. Solve for Mn.

Solution: Let oxidation number of Mn = x. $x + 4(-2) = -1 \Rightarrow x - 8 = -1 \Rightarrow x = +7$.

21.(a) **Hint:** It is the salt of a strong acid (HCl) and a weak base (NH_4OH). Consider cationic hydrolysis.

Solution: NH_4Cl is the salt of a strong acid and a weak base. The NH_4^+ ion hydrolyses to give H_3O^+, so the solution is acidic (pH < 7).

22.(c) **Hint:** At the cathode, the ion with the higher (less negative) reduction potential is discharged. Compare Cu^{2+} with H^+.

Solution: Cu^{2+} is more easily reduced than H^+ (higher reduction potential), so copper metal is deposited at the cathode: $Cu^{2+} + 2e^- \rightarrow Cu$.

23.(a) **Hint:** CO_2 is a linear molecule (O = C = O). The central carbon has two sigma bonds and no lone pairs.

Solution: In CO_2 (O = C = O), carbon forms 2 sigma bonds with no lone pair (2 electron domains) → sp hybridisation, giving a linear shape.

24.(a) **Hint:** In an endothermic reaction, heat is absorbed, so the products have more enthalpy than the reactants.

Solution: An endothermic reaction absorbs heat, so the enthalpy of products > enthalpy of reactants, giving ΔH positive (+).

25.(c) **Hint:** The gas must be chemically unreactive so it does not react with the hot filament. Look among the noble gases.

Solution: Argon, a noble (inert) gas, is used to fill electric bulbs because it does not react with the hot tungsten filament.

26.(b) **Hint:** The most abundant element by mass in the crust is a non-metal present in most rocks and minerals (silicates, oxides).

Solution: Oxygen is the most abundant element by mass in the Earth's crust (≈ 46%), followed by silicon.

27.(b) **Hint:** The coinage metals are the three Group 11 elements traditionally used to make coins.

Solution: The coinage metals are copper, silver and gold (Group 11). Hence copper is the coinage metal among the options.

28.(b) **Hint:** Distinguish this from roasting (heating in a plentiful supply of air). Here air is largely excluded.

Solution: Calcination is the strong heating of an ore in the absence or limited supply of air, driving off volatile matter (e.g. moisture, CO_2). (Roasting is done in excess air.)

29.(c) Given function is $f(x) = \sin\pi x$

So, period $= \frac{2\pi}{\pi} = 2$

30.(b) Given $f(x) = 10^x + 10^{-x}$

So, $f(-x)\ 10^{-x} + 10^{-(-x)}$

$= 10^{-x} + 10^x = 10^x + 10^{-x} = f(x)$

∴ f(x) is even

31.(b) Given equation is $x^2 - 5x + 4 = 0$

Here, discriminant $= (-5)^2 - 4.1.4 = 25 - 16 = 9$ which is perfect square.

∴ Roots are rational and unequal.

32.(d) As the matrix $\begin{pmatrix} 6 & x-2 \\ 3 & x \end{pmatrix}$ has no inverse

So $\begin{vmatrix} 6 & x-2 \\ 3 & x \end{vmatrix} = 0$

or, $6x - 3(x - 2) = 0$

or, $6x - 3x + 6 = 0$

or, $3x = -6$ ∴ $x = -2$

33.(c) Here no. of points = 10 no. of collinear points = 4

∴ No. of triangles = C(10, 3) – C(4, 3) = 116

34.(b) $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$

or, $\frac{\pi}{2} - \cos^{-1}x + \frac{\pi}{2} - \cos^{-1}y = \frac{2\pi}{3}$

or, $\cos^{-1}x + \cos^{-1}y = \pi - \frac{2\pi}{3}$

∴ $\cos^{-1}x + \cos^{-1}y = \frac{\pi}{3}$

35.(c) $\vec{i}.(\vec{j} \times \vec{k}) + \vec{j}.(\vec{k} \times \vec{i}) + \vec{k}.(\vec{i} \times \vec{j})$

$= \vec{i}.\vec{i} + \vec{j}.\vec{j} + \vec{k}.\vec{k}$

$= 1 + 1 + 1 = 3$

36.(a) Given equation is $kx^2 - 4xy + 5y^2 = 0$

As the lines are coincident so $(-2)^2 = k \times 5$

i.e. $k = \frac{4}{5}$

37.(c) Here $\frac{1}{2}, \frac{1}{3}$, k are direction cosines

So, $\left(\frac{1}{2}\right)^2 + \left(\frac{1}{3}\right)^2 + k^2 = 1$

i.e. $k^2 = 1 - \frac{1}{4} - \frac{1}{9} = \frac{23}{36}$

38.(a) Here p is true, q is false.

So $p \wedge q$ is false and $(p \wedge q) \Rightarrow r$ is true.

39.(c) $\sin^{11}x \cos^{14}x$ is odd

So, $\int_{-a}^{a} \sin^{11}x \cos^{14}x\, dx = 0$

40.(c) $\lim\limits_{x \to 0} \frac{\sin x^0}{x} = \lim\limits_{x \to 0} \frac{\sin \frac{\pi x}{180}}{\frac{\pi x}{180}} \times \frac{\pi}{180} = \frac{\pi}{180}$

41.(a) Put $y = \frac{1}{x}$

So $dy = -\frac{1}{x^2}dx$

Here $dx = 1 - 0.98 = -0.02$

Then $dy = -\frac{1}{12} \times -0.02 = 0.02$

42.(d) Here y-axis makes an angle of 90° with x-axis so slope = $\tan 90° = \infty$

43.(b) Given $y = \log_{\sqrt{x}} x = 2\log_x x = 2$

$\therefore \quad \frac{dy}{dx} = 0$

44.(d) We have, $\int e^x\{f(x) + f'(x)\}\, dx = e^x f(x) + c$.

So, $f(x) = \tan x$ and hence $f'(x) = \sec^2 x$

45.(b) When a coin is tossed twice then S = {HH, HT, TH, TT}

So the required probability $= \frac{1}{4}$

46.(c) Given regression equation of y on x is

$3x + 5y = 15$

or, $5y = -3x + 15$

i.e. $y = -\frac{3}{5}x + 3$

So, $b_{yx} = -\frac{3}{5}$

47.(d) Given differential equation is $\frac{d^2y}{dx^2} = \sqrt[3]{1 + \left(\frac{dy}{dx}\right)^4}$

i.e. $\left(\frac{d^2y}{dx^2}\right)^3 = 1 + \left(\frac{dy}{dx}\right)^4$

So degree = 3

48.(b) $\int \log_e x\, dx = \int 1 \log_e x\, dx$

$= \log_e x \int dx - \int\left(\frac{d}{dx}\log_e x \int dx\right)dx$

So, $\int_1^e \log_e x\, dx = [x\log_e x - x]_1^e$

$= (e\log_e e - e) - (1\log 1 - 1)$

$= (e - e) - (0 - 1) = 1$

49.b	**50.c**	**51.b**	**52.b**	**53.a**	**54.c**
55.d	**56.d**	**57.b**	**58.b**	**59.a**	**60.b**

Section – II

61.(a) KE will be least at maximum height where range $= \frac{R}{2}$

62.(c) $\frac{\left(\frac{d\theta}{dt}\right)_1}{\left(\frac{d\theta}{dt}\right)_2} = \frac{\theta_1 - \theta_0}{\theta_2 - \theta_0}$

or, $\frac{\frac{20}{7}}{\frac{12}{t}} = \frac{50 - 10}{34 - 10}$

or, $\frac{20t}{12 \times 7} = \frac{50 - 10}{34 - 10}$

or, $\frac{20t}{12 \times 7} = \frac{40}{24}$

or, $t = 7$ min

63.(c) First case

$\frac{Q}{t} = \frac{KAd\theta}{l} = \frac{m_1 L_f}{t}$

or, $\frac{KAd\theta}{l} = 0.1 \times L_f$ .... (1)

2nd case

$\frac{Q}{t} = \frac{K4Ad\theta \times 2}{4 \times l} = \frac{m_2}{t} L_f$ .... (2)

Dividing (2) by (1)

$\frac{m_2}{t} = 2 \times 0.1 = 0.2$ g/s

64.(a) $f' = \frac{v + v_0}{v - v_s} \times f$

or, $\frac{f'}{f} = \frac{v + \frac{v}{2}}{v - \frac{v}{2}} = \frac{3v}{2} \times \frac{2}{v} = 3$

% increase $= \left(\frac{f'}{f} - 1\right) \times 100\%$

$= (3 - 1) \times 100\%$

$= 200\%$

65.(b) $\frac{2f}{f} = \sqrt{\frac{T}{g}}$

or, $T = 36$ kg ωt

66.(a) When one end of rod coincide with image then that end lies at C so

$u_1 = v_1 = 2f$

Diminished image means object lies beyond C

for end B

$u_2 = 2f + \frac{f}{2} = \frac{5f}{2}$

$v_2 = \frac{fu_2}{u_2 - f} = \frac{f \times \frac{5f}{2}}{\frac{5f}{2} - f} = \frac{5f^2}{2} \times \frac{2}{3f} = \frac{5f}{3}$

Length of image (I) $= V_1 - V_2 = 2f - \frac{5f}{3} = \frac{f}{3}$

67.(c) Total = 50 cm

In air distance = 40 cm

Real distance $= \frac{40}{\mu}$

$= \frac{40}{4} \times 3 = 30$ cm

$\therefore$ Actual distance = 10 + 30 = 40 cm

68.(d) $0 = \sqrt{I_1 I_2} = \sqrt{4 \times 16} = 8$ cm

69.(a) $\beta = \frac{D\lambda}{d}$

$$\Delta\beta = \frac{\lambda \Delta D}{d}$$

or, $$\lambda = \frac{\Delta\beta \times d}{\Delta D} = \frac{3 \times 10^{-5} \times 10^{-3}}{5 \times 10^{-2}}$$

$$= 6 \times 10^{-7}\ \text{m} = 6000 \times 10^{-10}\ \text{m}$$

$$= 6000\ \text{Å}$$

70.(a) At. surface of outer sphere

$$V = \frac{Q}{4\pi\varepsilon_0 2R} + \frac{2Q}{4\pi\varepsilon_0 2R}$$

$$= \frac{3Q}{4\pi\varepsilon_0 2R}$$

When sphere are connected by wire then all charge lies on outer sphere

$$V' = \frac{3Q}{4\pi\varepsilon_0 2R} = V$$

71.(c) Total energy stored = energy stored in combination

or, $$\left(\frac{1}{2} \times 10^{-6} \times 500^2\right) n = \frac{1}{2} \times 10^{-6} (3000)^2$$

or, $$n = \left(\frac{3000}{500}\right)^2 = 36$$

72.(d) Ladder are at same potential so act as open circuit so $R_{eq} = \frac{3R}{2}$

73.(d) $$\frac{N}{N_0} = 80\% = \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

or, $$\frac{4}{5} = \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

or, $$ln\left(\frac{4}{5}\right) = \frac{10}{T_{1/2}} ln\left(\frac{1}{2}\right)$$

or, $T_{1/2} = 31$ days

$$\frac{N'}{N_0} = \left(\frac{1}{2}\right)^{\frac{t_2}{T_{1/2}}} = \left(\frac{1}{2}\right)^{\frac{20}{31}} = 0.639$$

$$\%\ \text{left} = \frac{N'}{N_0} \times 100\% = 0.639 \times 100\%$$

$$= 63.9\%$$

74.(b) **Hint:** The chief ore of zinc is its sulphide. (Haematite is an iron ore, bauxite an aluminium ore, cinnabar a mercury ore.)

Solution: The chief ore of zinc is zinc blende (sphalerite), ZnS. It is roasted to ZnO and then reduced to obtain zinc.

75.(c) **Hint:** Alkynes contain one carbon–carbon triple bond, so they have four fewer hydrogens than the corresponding alkane.

Solution: Alkynes have one C≡C triple bond and the general formula C_nH_{2n-2} (e.g. ethyne C_2H_2).

76.(b) **Hint:** Ethene has a C=C double bond. Bromine adds across the double bond, so the reagent is consumed and decolourised.

Solution: Br_2 adds across the C=C double bond of ethene (electrophilic addition) to give 1,2-dibromoethane, decolourising the bromine water.

77.(c) **Hint:** Freons are chlorofluorocarbons (CFCs) containing both chlorine and fluorine.

Solution: Dichlorodifluoromethane, CCl_2F_2 (Freon-12), is a chlorofluorocarbon used as a refrigerant.

78.(b) **Hint:** Primary alcohols oxidise in two stages. Identify the first product before further oxidation to an acid.

Solution: A primary alcohol is first oxidised to an aldehyde, which on further oxidation gives a carboxylic acid. (Secondary alcohols give ketones.)

79.(a) **Hint:** This is an esterification (Fischer esterification); concentrated H_2SO_4 acts as a catalyst and dehydrating agent.

Solution: Acetic acid + ethanol — (conc. H_2SO_4) → ethyl acetate (an ester) + water.

$CH_3COOH + C_2H_5OH \rightarrow CH_3COOC_2H_5 + H_2O$.

80.(c)

$$2R - O - H \xrightarrow[170°C]{\text{conc. } H_2SO_4} R - C = C - R^{|} + H_2O$$

81.(b)

82.(a) Here n(A) + n(B) = 65 + 95 = 160 > 130 = n(U)

So maximum possible value of $n(A \cup B) = n(U) = 130$

83.(d) Here $S_\infty = 15$, a = 5

So $S_\infty = \frac{a}{1-r}$ or, $15 = \frac{5}{1-r}$ or, $1 - r = \frac{5}{15}$

or, $r = 1 - \frac{1}{3}$ $\therefore$ $r = \frac{2}{3}$

84.(c) Given $z = r(\cos\theta + i\sin\theta)$

So $iz = r(i\cos\theta - \sin\theta)$

$$= r\left[\cos\left(\frac{\pi}{2} + \theta\right) + i\sin\left(\frac{\pi}{2} + \theta\right)\right]$$

$\therefore$ $\arg(iz) = \frac{\pi}{2} + \theta$

85.(a) Given word is NUMBER

For the arrangements that begin and end with vowel, U and E are kept fixed and remaining 4 letters are arranged in 4! ways. But U and E can interchange the position, so total no. of arrangements = 2 × 4! = 48.

86.(d) Given series is $1 + \frac{a+bx}{1!} + \frac{(a+bx)^2}{2!} + ...$

$$= e^{a+bx}$$

$$= e^a e^{bx} = e^a \sum_{n=0}^{\infty} \frac{(bx)^n}{n!} = e^a \sum_{n=0}^{\infty} \frac{b^n x^n}{n!}$$

So, coefficient of $x^{10} = \frac{e^a b^{10}}{10!}$

87.(c) Given, coefficient of x^r = coefficient of x^{r+1}

or, C(2n + 1, r) = C(2n + 1, r + 1)

So, $r = r + 1 \Rightarrow 1 = 0$, not possible or, $r + r + 1 = 2n + 1 \Rightarrow 2r = 2n \quad \therefore r = n$

88.(b) Given $\frac{a}{\cos A} = \frac{b}{\cos B}$

or, $\frac{2R\sin A}{\cos A} = \frac{2R\sin B}{\cos B}$

or, $\tan A = \tan B$

or, $A = B$

$\therefore$ Δ is isosceles

89.(b) Given circle is $x^2 + y^2 = 16$

$\therefore$ Length of tangent drawn from (4, 3) to the circle

$\sqrt{x_1^2 + y_1^2 - 16} = \sqrt{4^2 + 3^2 - 16}$

$= \sqrt{16 + 9 - 16} = \sqrt{9} = 3$ units

90.(b) Given parabola is $y^2 = 16x$.

So $a = 4$ and line is $y = 2x + c$

So $m = 2$, $c = c$

Thus line touches parabola if $c = \frac{a}{m} = \frac{4}{2} = 2$

91.(d) Given $x = a\cos\theta$, $y = b\sin\theta$

So $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = \cos^2\theta + \sin^2\theta$

i.e. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, which an ellipse

92.(b) Given $x^2 - y^2 = a^2$, $xy = c^2$ which are rectangular hyperbola.

So $e = \sqrt{2}$, $e' = \sqrt{2}$. Now, $e^2 + e'^2 = 2 + 2 = 4$

93.(a) Equation of plane parallel to $x + 2y + 2z + 7 = 0$ and passing through the point (1, 2, 2) is

$(x - 1) + 2(y - 2) + 2(z - 2) = 0$

i.e. $x + 2y + 2z = 9$

94.(c) Given $y = ax^2 - 12x + 20$

So $\frac{dy}{dx} = 2ax - 12$

As the function has local minimum value at $x = 2$.

So $2a \times 2 - 12 = 0$

or, $4a = 12 \quad \therefore a = 3$

95.(b) Given $y = A\sin x + B\cos x$

So $\frac{dy}{dx} = A\cos x - B\sin x$ and $\frac{d^2y}{dx^2} = -A\sin x - B\cos x$

$= -(A\sin x + B\cos x) = -y$

$\therefore \frac{d^2y}{dx^2} + y = 0$

96.(a) Given curve is $y = 2x - x^2$.

Along x-axis, $y = 0$

So $2x - x^2 = 0$

i.e. $x = 0, 2$

$\therefore$ Required area $= \int_0^2 y\,dx = \int_0^2 (2x - x^2)\,dx$

$= \left[x^2 - \frac{x^3}{3}\right]_0^2 = \left(2^2 - \frac{2^3}{3}\right) - (0 - 0)$

$= 4 - \frac{8}{3} = \frac{4}{3}$

97.c **98.d** **99.b** **100.a**

…The End…