

Section - I

- 1.(c) $\vec{A} \cdot \vec{B} = 0$ means $\vec{A} \perp \vec{B}$, $\vec{A} \cdot \vec{C} = 0$ means $\vec{A} \perp \vec{C}$ then $(\vec{B} \times \vec{C})$ is \perp to \vec{B} and \vec{C} , i.e. parallel to \vec{A} .
- 2.(a) $F = \frac{dp}{dt} = a + 2bt$
When $t = 0$, $F = a$
- 3.(b) $\Delta PE = -\frac{GMm}{r} - \left(-\frac{GMm}{R}\right)$
 $= gMm \left[\frac{1}{R} - \frac{1}{R+2R} \right]$
 $= 9R^2m \times \frac{2}{3R} = \frac{2}{3} mgR$
- 4.(a) Dimension of body increases internally as well as externally on increasing temperature.
- 5.(b) $\frac{P_1}{P_2} = \frac{\sigma A_1 T^4}{\sigma A_2 T^4} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{1}{2}\right)^2 = 1:4$
- 6.(d) $\frac{f_0}{f_c} = \frac{f_0}{(2p-1)f_0} = \frac{p \times v \times 4l}{(2p-1)2l \times v} = \frac{2p}{2p-1}$
- 7.(c) $\frac{V'}{V} = (n)^{2/3}$
or, $V' = (64)^{2/3} V = 16V$
- 8.(a) $R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$
or, $0.1 = \frac{0.5R_2}{0.5 + R_2}$
or, $0.05 + 0.1R_2 = 0.5R_2$
or, $R_2 = \frac{0.05}{0.4} = 0.125\Omega$
- 9.(a) $M' = \sqrt{M^2 + 2M^2 \cos 120^\circ + M^2}$
 $= \sqrt{M^2 - M^2 + M^2} = M$
- 10.(a) $f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{L'C'}}$
or, $LC = \frac{L}{2} C'$
or, $C' = 2C$
- 11.(d) $\mu = \frac{\text{Real depth}}{\text{Apparent depth}}$
 $= \frac{40}{30} = \frac{4}{3}$
- 12.(d) $\frac{I_{\max}}{I_{\min}} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2 = \left(\frac{2+1}{2-1}\right)^2 = 9:1$
- 13.(b) Maximum energy is independent of intensity of incident radiation.
- 14.(a) Diode is reverse biased so
 $I = 0$
- 15.(a)
- 16.(d) $+5 \quad +2$
 $\text{HNO}_3 \rightarrow \text{NO}$
Since 2 mole $\text{NO} \downarrow$ +ve charge reduce A.
- 17.(b) $K_p = K_c \cdot (RT)^{-2-1}$
 $\therefore K_p > K_c$
- 18.(d) $2\text{H}_2\text{O} \rightarrow 2\text{H}_2 + \text{O}_2$
2:1
- 19.(a)
- 20.(c)

- 21.(d)
- 22.(c)
- 23.(b) H^+ comes from H_2SO_4 is accepted by HNO_3 to form
- $$\begin{array}{c} + \\ \text{H} - \text{O} - \text{NO}_2 \\ | \\ \text{H} \end{array}$$
- 24.(d)
- 25.(b)
- 26.(b)
- 27.(a)
- 28.(d)
- 29.(b) We have $\sim(p \wedge q) \equiv (\sim p) \vee (\sim q)$
So the negation of the statement is $2 \times 3 \neq 6$ or 2 is not a prime number.
- 30.(c) $(A - B)' = (A \cap B)' = A' \cup (B')' = A' \cup B$
- 31.(d) Since the equations have both common roots so we have $\frac{1}{1} = \frac{a}{c} = \frac{b}{d}$
Taking last two ratios, we have $ad = bc$
- 32.(b) The equation $ax^2 + bx + c = 0$ is quadratic only when the coefficient of x^2 is not zero. So we must have $a \neq 0$.
- 33.(b) Here $D = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -1 - 1 = -2 \neq 0$
 \therefore It has only one solution.
- 34.(d) Here the triangle formed by the given sides is right angled triangle and right angle at origin. So the orthocentre is the midpoint of hypotenuse.
Putting $x = 0$, $y = 8$ and $y = 0$, $x = 6$.
So circumcentre = $\left(\frac{0+6}{2}, \frac{8+0}{2}\right) = (3, 4)$
- 35.(c) The line $lx + my + n = 0$ is a normal to the circle $x^2 + y^2 = a^2$ if it passes through the centre. So $l \times 0 + m \times 0 + n = 0$. i.e., $n = 0$
- 36.(d) Here $4^2 > 3 \times 5$ i.e., $16 > 15$
So, the point is external and hence two tangents are possible.
- 37.(c) Given parabola is $y^2 = 12x$. So $a = 3$
Let $P(x, y)$ be any point on the parabola. Then focal distance = $x + a = x + 3$. So $x + 3 = 4$
i.e., $x = 1$
- 38.(d) Length of the segment = $\sqrt{6^2 + 2^2 + 3^2}$
 $= \sqrt{36 + 4 + 9} = \sqrt{49} = 7$ units
- 39.(c) Length of perpendicular from origin
 $= \pm \frac{2 \times 0 + 2 \times 0 + 6}{\sqrt{1^2 + 2^2 + 2^2}}$
 $= \frac{6}{3} = 2$ units
- 40.(b) Let θ be the angle between the vectors.
Then $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$
 $\Rightarrow |\vec{a}| |\vec{b}| \cos \theta = |\vec{a}| |\vec{b}| \sin \theta$
 $\Rightarrow \tan \theta = 1$
 $\Rightarrow \theta = 45^\circ$
- 41.(b) Given $A = \tan^{-1}x$ i.e., $x = \tan A$

So, $\sin 2A = \frac{2 \tan A}{1 + \tan^2 A} = \frac{2x}{1+x^2}$

- 42.(c) No. of faced cards = 12
 No. of red cards = 26
 No. of red faced cards = 6
 So probability of getting a red faced card = $\frac{6}{52} = \frac{3}{26}$

- 43.(d) Given regression equation is
 $2x + 3y + 5 = 0$
 or, $3y = -2x - 5$
 i.e. $y = -\frac{2}{3}x - \frac{5}{3}$
 \therefore Regression coefficient of y on x = $-\frac{2}{3}$

44.(c) Given, $\lim_{x \rightarrow 0} \frac{\log_e(1+x)}{x}$
 $= \lim_{x \rightarrow 0} \log_e e^{\frac{\log_e(1+x)}{x}}$
 $= \log_e e \times 1 = \log_e e$

- 45.(a) Since the function is continuous at $x = 1$, so we must have
 $\lim_{x \rightarrow 1^-} f(x) = f(1) = \lim_{x \rightarrow 1^+} f(x)$
 Then $\lim_{x \rightarrow 1^-} -2x + 1 = p$
 $\Rightarrow 2 \times 1 + 1 = p \Rightarrow p = 3$

46.(b) Given $y = \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$
 or, $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots - 1$
 or, $y = e^x - 1$
 $\therefore \frac{dy}{dx} = e^x = y + 1$

- 47.(c) Given $f(x) = \cot^{-1} x$
 So $f'(x) = -\frac{1}{1+x^2} < 0$ for all real x.
 So the function is strictly decreasing.

48.(d) Given $y \sin x = x + c$
 On differentiation, $y \frac{d \sin x}{dx} + \sin x \frac{dy}{dx} = \frac{dx}{dx} + \frac{dc}{dx}$
 or, $y \cos x + \sin x \frac{dy}{dx} = 1$
 or, $\frac{dy}{dx} + \cot x y = \operatorname{cosec} x$
 $\frac{dy}{dx} + \cot x y = \operatorname{cosec} x$

On comparing with $\frac{dy}{dx} + Py = Q$ we get

$P = \cot x$

- 49.(c) 50.(b) 51.(a) 52.(d) 53.(a) 54.(c)
 55.(d) 56.(b) 57.(d) 58.(b) 59.(c) 60.(c)

Section - II

- 61.(c) He walk last 8m in 8 sec so

Remaining distance = $18 - 8 = 10\text{m}$
 in each time he walk only 2m in 14 sec

So no of times = $\frac{10}{2} = 5$ times
 \therefore Total time = $5 \times 14 + 8 = 78$ sec

62.(a) $\omega = \omega_0 + \alpha t$
 or, $0 = 2\pi f + \alpha t$
 or, $\alpha = -\frac{2\pi \cdot 900}{60 \cdot 60} = -\frac{\pi}{2} \text{ rad/s}^2$

63.(d) $PA = 2\pi r T$
 $\rho g h \times \pi r^2 = 2\pi r T$
 or, $h = \frac{2T}{\rho g r} = \frac{2 \times 0.075 \times 2}{1000 \times 10 \times 0.1 \times 10^{-3}} = 0.3\text{m}$

64.(c) $40\% \text{ of } \frac{1}{2} m v^2 = m s d \theta$
 or, $d\theta = \frac{0.4 \times 500^2}{2 \times 0.03 \times 4200} = 396^\circ\text{C}$

65.(c) $r = \frac{P}{\rho T} = \frac{10^5}{1.775 \times 300} = 188 \text{ Jkg}^{-1}\text{K}^{-1}$
 $C_p = C_v + r = 658 + 188 = 846 \text{ Jkg}^{-1}\text{K}^{-1}$

66.(a) For convex lens
 $V = \frac{fu}{u-f} = \frac{20 \times 25}{25-20} = \frac{20 \times 25}{5} = 100 \text{ cm}$

So light incident normally on convex mirror so
 $r = (100 - 40) = 60 \text{ cm}$
 $f = \frac{r}{2} = \frac{60}{2} = 30 \text{ cm}$

67.(a) $\beta = \frac{D\lambda}{d}$
 or, $\Delta\beta = \frac{\lambda}{d} \Delta D$
 or, $\lambda = \frac{\Delta\beta d}{\Delta D} = \frac{3 \times 10^{-5} \times 10^{-3}}{5 \times 10^{-2}} = 6000 \text{ \AA}$

68.(a) $\vec{E}_x = -\frac{\partial v}{\partial x} = -5\hat{i}$
 $\vec{E}_y = -\frac{\partial v}{\partial y} = -4\hat{j}$
 $\vec{E}_z = -\frac{\partial v}{\partial z} = +6\hat{k}$
 $\therefore E = \vec{E}_x + \vec{E}_y + \vec{E}_z = -5\hat{i} - 4\hat{j} + 6\hat{k}$

- 69.(c) Antinode is formed at 20 cm

So $\frac{\lambda}{4} = 20\text{cm}$

$\lambda = 80 \text{ cm}$

no of loops (n) = $-\frac{l}{\lambda/2} = \frac{2 \times 2}{0.8} = 5$

$f = n f_0 = n \times \frac{1}{2l} \sqrt{\frac{Tl}{M}}$
 $= 5 \times \frac{1}{2 \times 2} \sqrt{\frac{20 \times 2}{10^{-3}}} = 250 \text{ Hz}$

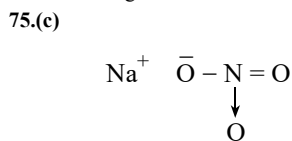
70.(d) $l = 2\pi R$
 $R = \frac{l}{2\pi} = \frac{2}{2\pi} = \frac{1}{\pi} \text{ m}$
 $M = IA$
 $= 1 \times \pi R^2 = \pi \left(\frac{1}{\pi}\right)^2 = \frac{1}{\pi} \text{ Am}^2$

71.(c) $E_2 = M \frac{dI_1}{dt}$
 $= \frac{5(5-0)}{10^{-3}} = 25000 \text{ V} = 25 \text{ KV}$

72.(a) $2E - E = \frac{hc}{\lambda}$
 or, $E = \frac{hc}{\lambda} \dots (1)$
 Again $\frac{4E}{3} - E = \frac{hc}{\lambda'}$
 or, $\frac{E}{3} = \frac{hc}{\lambda'} \dots (2)$
 From (1) & (2)
 $\frac{hc}{3\lambda} = \frac{hc}{\lambda'}$
 or, $\lambda' = 3\lambda$

73.(d) $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$
 or, $\left(1 - \frac{3}{4}\right) = \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$
 or, $\left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$
 or, $T_{1/2} = \frac{t}{2} = \frac{3}{4 \times 2} = \frac{3}{8} \text{ sec}$

74.(c) 28gm CO = 1 mole
 46gm C₂H₅OH = 1 mole
 36gm H₂O = 2 mole
 54gm N₂O₅ = 0.5 mole



76.(b) H₂SO₄ & NaOH
 $200 \times \frac{1}{10} \quad 100 \times \frac{1}{20}$
 $= 20 \text{ ml } 1\text{N} \quad 5 \text{ ml } 1\text{N}$
 Thus acid left 15 ml 1N H₂SO₄
 $\therefore \omega = \frac{15 \times 1 \times 49}{1000} = 0.735 \text{ gm}$

77.(d) pH = 5, $\text{H}^+ = \frac{10^{-5}}{1000} = 10^{-8} \text{ m acid}$
 $\therefore \text{pH} = -\log(10^{-8} + 10^{-7}) = 6.96$

78.(c)
79.(c)

80.(c)

81.(d)

82.(b) Given $y = f(x) = \frac{1}{4 - \sin 3x}$, $-1 \leq \sin 3x \leq 1$
 So, $-1 \leq -\sin 3x \leq 1$
 or, $4 - 1 \leq 4 - \sin 3x \leq 4 + 1$
 or, $3 \leq \frac{1}{y} \leq 5$

i.e. $\frac{1}{5} \leq y \leq \frac{1}{3}$ $\therefore \text{Range} = \left[\frac{1}{5}, \frac{1}{3}\right]$

83.(c) Given $\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + \dots$ to 'n' terms
 $= 1 - \frac{1}{2} + 1 - \frac{1}{4} + 1 - \frac{1}{8} + 1 - \frac{1}{16} + \dots$ to 'n' terms
 $= (1+1+\dots \text{ to } n \text{ terms}) - \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots \text{ to } n \text{ terms}\right)$
 $= n - \frac{1}{2} \left[\frac{1 - \left(\frac{1}{2}\right)^n}{1 - \frac{1}{2}} \right] = n - \left(1 - \frac{1}{2^n}\right) = n - 1 + 2^{-n}$

84.(c) $(1 + \omega)^7 = A + B\omega$
 or, $(-\omega^2)^7 = A + B\omega$
 or, $-\omega^{14} = A + B\omega$
 or, $\omega^{12}\omega^2 = -A - B\omega$
 or, $(\omega^3)^4 \omega^2 = -A - B\omega$
 or, $1 \cdot \omega^2 = -A - B\omega$
 or, $-1 - \omega = -A - B\omega$
 or, $A + B\omega = 1 + \omega$
 $\Rightarrow A = 1, B = 1$

85.(b) Given $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = \begin{vmatrix} 1+\omega+\omega^2 & \omega & \omega^2 \\ 1+\omega+\omega^2 & \omega^2 & 1 \\ 1+\omega+\omega^2 & 1 & \omega \end{vmatrix}$
 (Applying $C_1 \rightarrow C_1 + C_2 + C_3$)
 $= \begin{vmatrix} 0 & \omega & \omega^2 \\ 0 & \omega^2 & 1 \\ 0 & 1 & \omega \end{vmatrix} (\because 1 + \omega + \omega^2 = 0)$
 $= 0 (\because C_1 = 0)$

86.(d) $C(20, 2) - C(4, 2) + 1 = 185$
87.(b) Sum of the coefficients = 4096
 or, $2^n = 4096$
 or, $2^n = 2^{12}$
 $\therefore n = 12$

Here n is even, so the greatest coefficient = $C\left(n, \frac{n}{2}\right)$
 $= C(12, 6)$
 $= 924$

88.(a) Along y-axis, we have $x = 0$.
 Then $by^2 + 2fy + c = 0$
 For point of intersection $\frac{\partial f}{\partial x} = 0$
 $2ax + 2hy + 2g = 0$

Since $x = 0, y = -\frac{g}{h}$

Then $b\left(-\frac{g}{h}\right)^2 + 2f\left(-\frac{g}{h}\right) + c = 0$
 $\Rightarrow bg^2 + ch^2 = 2fgh$

89.(c) Eccentricity of first ellipse $= \sqrt{1 - \frac{25}{169}}$
 and eccentricity of second ellipse $= \sqrt{1 - \frac{b^2}{a^2}}$
 According to question, $\sqrt{1 - \frac{25}{169}} = \sqrt{1 - \frac{b^2}{a^2}}$
 or, $1 - \frac{25}{169} = 1 - \frac{b^2}{a^2}$
 or, $\frac{25}{169} = \frac{b^2}{a^2} \Rightarrow \frac{a}{b} = \frac{13}{5}$

90.(c) Given $|\vec{a}| = 5, |\vec{b}| = 12, \vec{a} \cdot \vec{b} = 36$
 So, $\vec{a} \cdot \vec{b} = 36$ or, $|\vec{a}| |\vec{b}| \cos\theta = 36$
 or, $5 \times 12 \cos\theta = 36$
 i.e. $\cos\theta = \frac{3}{5}$

Then $\sin\theta = \sqrt{1 - \cos^2\theta} = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$
 and $|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \sin\theta = 5 \times 12 \times \frac{4}{5} = 48$

91.(d) Given $\tan\left(\cos^{-1}\frac{3}{4} + \sin^{-1}\frac{3}{4} - \sec^{-1}3\right)$
 $= \tan\left(\frac{\pi}{2} - \sec^{-1}3\right) = \cot \sec^{-1}3$
 $= \cot \cot^{-1}\left(\frac{1}{\sqrt{3^2-1}}\right)$
 $= \frac{1}{\sqrt{9-1}} = \frac{1}{\sqrt{8}} = \frac{1}{2\sqrt{2}}$

92.(d) Given $y = 1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \dots, |x| > 1$
 $= \frac{1}{1 - \frac{1}{x}} = \frac{1}{\frac{x-1}{x}} = \frac{x}{x-1} \Rightarrow y = \frac{x}{x-1}$

Then $\frac{dy}{dx} = \frac{(x-1) \cdot 1 - x \cdot 1}{(x-1)^2} = \frac{y-1}{x-1} \dots (i)$
 $= \frac{x-1-x}{(x-1)^2} = -\frac{1}{(x-1)^2}$
 $= -\left(\frac{y}{x}\right)^2 = -\frac{y^2}{x^2}$

93.(c) Given $y^2 = px^3 + q$
 or, $2y \frac{dy}{dx} = p \cdot 3x^2$

i.e. $\frac{dy}{dx} = \frac{3px^2}{2y}$

At point (2, 3)

Slope of tangent $= \frac{3p \cdot 2^2}{2 \cdot 3} = 2p$

Also, eqⁿ of tangent is

$y = 4x - 5$, slope = 4

So, $2p = 4$ i.e. $p = 2$

Again, $3^2 = 2 \cdot 2^3 + q$

or, $9 = 16 + q \Rightarrow q = -7$

94.(b) $I = \int \cos y \cdot 2y \, dy$ Let $y = \sqrt{x}$
 $= 2 \int y \cos y \, dy \Rightarrow y^2 = x \quad 2y \, dy = dx$
 $= 2 \left[y \int \cos y \, dy - \int \left(\frac{dy}{dy}\right) \int \cos y \, dy \, dy \right]$
 $= 2[y \sin y - \int \sin y \, dy]$
 $= 2[y \sin y + \cos y] + c$
 $= 2[\sqrt{x} \sin \sqrt{x} + \cos \sqrt{x}] + c$

95.(b) Given $I = \int_0^1 \frac{dx}{e^x + e^{-x}}$
 $= \int_0^1 \frac{dx}{e^x + \frac{1}{e^x}} = \int_0^1 \frac{e^x \, dx}{(e^x)^2 + 1^2}$
 $= \left[\tan^{-1} e^x \right]_0^1 = \tan^{-1} e - \tan^{-1} 1$
 $= \tan^{-1} \frac{e-1}{1+e \cdot 1} = \tan^{-1} \frac{e-1}{e+1}$

96.(b) Given curves are

$y = x^2$

and $y = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

Solving $y = x^2$ with $y = x$, we get $x = 0, 1$

Solving $y = x^2$ with $y = -x$, we get $x = 0, -1$

Thus, the required area = $A_1 + A_2$

Where $A_1 = \int_0^1 (x - x^2) \, dx$ and $A_2 = \int_0^{-1} (-x - x^2) \, dx$
 $= \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_0^1 - 0 = -\left[\frac{x^2}{2} + \frac{x^3}{3} \right]_0^{-1}$
 $= \left(\frac{1}{2} - \frac{1}{3} \right) = -\left[\frac{1}{2} - \frac{1}{3} \right] + 0$
 $= \frac{3-2}{6} = -\frac{3-2}{6}$
 $= \frac{1}{6} = -\frac{1}{6}$
 $= \frac{1}{6}$ (Area positive)

\therefore Required area $= \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$ sq. units.

97.(c)

98.(c)

99.(c)

100.(c)

....Best of Luck...