Section – I

1.(a) $\frac{(\Delta p)_1}{(\Delta P)_2} = \frac{r_2}{r_1} = \frac{1}{\frac{r_1}{r_2}} = \frac{1}{\frac{1}{3}} = 3:1$

2.(b) $P = \frac{w}{t} = \frac{mgh}{t} = \frac{mgs\sin\theta}{t} = mg\,v\sin\theta$

3.(b) <u>For stone</u>

$h = \frac{1}{2}gt_1^2$ or, $t_1 = \sqrt{\frac{2h}{g}}$

<u>For sound</u>

$t_2 = \frac{h}{v}$

$\therefore\ t = t_1 + t_2 = \sqrt{\frac{2h}{g}} + \frac{h}{v}$

4.(a) When electrons are removed then +ve charge is developed so charge

(Q) = + ne = $10^{14} \times 1.6 \times 10^{-19}$

= 1.6×10^{-5}C = 16μC

5.(b) $R = \frac{\rho l}{\frac{\pi d^2}{4}}$

For 2nd wire $R' = \frac{\rho 4l}{\frac{\pi (2d)^2}{4}} = \frac{\rho l}{\frac{\pi d^2}{4}} = R$

6.(c) $\frac{E_A}{E_S} = \frac{l_A}{l_S}$

or, $E_A = \frac{75}{50} \times 1.02 = 1.53V$

7.(d) $\frac{B_{centre}}{B_{axis}} = \frac{\mu_0 NI/2a}{\frac{\mu_0 NIa^2}{2(3a^2 + a^2)^{3/2}}} = \frac{1}{2a} \times \frac{2 \times 8a^3}{a^2} = 8:1$

8.(a) $eV = hf_{max}$

or, $f_{max} = \frac{eV}{h} = \frac{1.6 \times 10^{-19} \times 42000}{6.62 \times 10^{-34}} = 1 \times 10^{19}$

9.(a) $\phi = B.A = \frac{F}{Il}A$

$= \frac{MLT^{-2}L^2}{AL} = ML^2T^{-2}A^{-1}$

10.(b) $R_{max} = r = \frac{u^2}{g} = \frac{10^2}{10} = 10m$

$\therefore\ A = \pi r^2 = \pi \times 10^2 = 100\pi$

11.(b) $\frac{du}{dQ} = \frac{Cv}{Cp} = \frac{1}{\gamma} = \frac{3}{5}$

12.(a) $F = I(\vec{l} \times \vec{B}) = I(\hat{i} \times \hat{j}) = \hat{k}$ = upward

13.(d) ${}_2\mu_1 = \frac{2C}{C} = 2$

$\sin C = \frac{1}{{}_2\mu_1} = \frac{1}{2} = \sin 30^o$

$C = 30^o$

14.(b) $\frac{hc}{\lambda} = E_4 - E_1$

or, $\lambda = \frac{hc}{E_4 - E_1}$

$= \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{\left(\frac{-13.6}{16} + 13.6\right)1.6 \times 10^{-19}}$

$= 9.7 \times 10^{-8}m = 970 \times 10^{-10}m$

= 970Å

15.(b) $E = \frac{W \times 1000}{V \times N} = \frac{0.45 \times 1000}{20 \times 0.5} = 45$

Basicity = (90/45) = 2

16.(b) In oxalate change in O.N. is 2 and MnO_4^- is 5.

2 moles of MnO_4^- is reduced by 5 moles of oxalate

2.5 moles

17.(d) To have magnetic quantum number -3, the value of *l* should be 3 and principal quantum number should be 4.

18.(c) Modern periodic table is arranged on the basis of atomic number so it has no space for isotopes as isotopes have same atomic number.

19.(d) H_2 and H^- differs by a proton so it represents conjugate acid-base pair.

20.(b) Washing soda(Na_2CO_3) removes Ca^{++} and Mg^{++} ions as insoluble carbonates.

21.(a) In alumino-thermic process metal oxides are reduced by aluminium powder.

22.(c) Zn dissolves in excess of NaOH solution forming sodium zincate(Na_2ZnO_2).

23.(b) Silver nitrate($AgNO_3$) is commonly known as lunar caustic.

24.(a) Addition of HCl to ethylene starts with adding H^+ to give a carbocation so it is electrophilic addition.

25.(b) For alkanes, number of isomers = $2^{n-4}+1$
$= 2^{6-2}+1=5$

26.(b) Lewisite

$$HC \equiv CH + AsCl_3 \longrightarrow \underset{\text{(Lewisite)}}{CHCl = CHAsCl_2}$$

27.(a)

$$CH \equiv CH \xrightarrow[NH_4OH]{AgNO_3} \underset{\text{(Ethylene glycol)}}{\underset{|\quad\;\; |}{CH_2 - CH_2} \atop {OH \quad OH}}$$

28.(a)

29.(b) For $l$nx, $x \in (0, \infty)$ but $l\text{n}x \neq 0$ in f(x)
So, $x \neq 1$

30.(c) It's simply the definition

31.(c) $\dfrac{-6-10}{4} = -4$

32.(a) $(1-\alpha\beta)x^2 - (\alpha+\beta)x + \alpha\beta = 0$
Discriminant $= (\alpha+\beta)^2 - 4\alpha\beta(1-\alpha\beta)$
$= (\alpha-\beta)^2 + (2\alpha\beta)^2 \geq 0$

33.(a) $4\left(1-\dfrac{9x^2}{16}\right)^{1/2}$ is valid for $\dfrac{9x^2}{16} < 1$
$\Rightarrow \quad x^2 < \dfrac{16}{9} \Rightarrow |x| < \dfrac{4}{3} \Rightarrow -\dfrac{4}{3} < x < \dfrac{4}{3}$

34.(d) $\displaystyle\int \frac{1+\sin x}{\cos^2 x}\,dx = \int \sec^2 x\,dx + \int \sec x \tan x\,dx$
$= \tan x + \sec x + c$

35.(c) $\displaystyle\lim_{x\to\infty}\left(\sqrt{\frac{1}{x^6}+1} - \left(\frac{1}{x^3}+1\right)\right) = 1 - 1 = 0$

36.(b) In the immediate nbhd of -5, $2 - x > 0$.
So, $f(x) = 2 - x \Rightarrow f'(x) = -1 \Rightarrow f'(-5)$
$= -1$

37.(d) $\vec{a}$ & $\vec{b}$ have same direction. So, $\vec{a} \times \vec{d}$ and $\vec{d} \times \vec{b}$ have opposite direction.

38.(c) Coefficient of y = 0, i.e. $-3 + k = 0 \Rightarrow k = 3$

39.(d) $r = \sqrt{b_{xy} \cdot b_{yx}}$
$= \sqrt{1.36 \times 0.8} = 1.04 > 1$
= Not possible

40.(c) $\log_N 2 + \log_N 3 + \ldots \log_N m$
$= \log_N (2.3\ldots. m) = \log_{m!} m! = 1$

41.(b) z = iy where y > 0
$\therefore \quad \arg(z) = \dfrac{\pi}{2}$

42.(a) ${}^{15}c_3 + {}^{15}c_{13} = {}^{15}c_3 + {}^{15}c_2 = {}^{16}c_3$

43.(d) Obvious.

44.(d) xy = 2 rectangular hyperbola

45.(b) n = 6, m = 4
$P(E) = \dfrac{4}{6} = \dfrac{2}{3}$

46.(b) For $\dfrac{\pi}{2} < x < \pi$, then $\cos x < 0$
So, $f(x) = -\cos x$
$\therefore \quad f'(x) = \sin x$
$\therefore \quad f'\left(\dfrac{3\pi}{4}\right) = \dfrac{1}{\sqrt{2}}$

47.(b) $y = \tan^{-1}x \qquad \therefore\ dy = \dfrac{1}{1+x^2}\,dx$
$\therefore \quad I = \int y^3 dy = \dfrac{(\tan^{-1}x)^4}{4} + c$

48.(a) d.c's of z-axis are 0, 0, 1
$\text{Proj}^n = (3-1).0 + (4-2).0 + (7-3).1 = 4$

49.(c)	50.(c)	51.(d)	52.(b)
53.(b)	54.(c)	55.(c)	56.(d)
57.(a)	58.(c)	59.(b)	60.(a)

Section – II

61.(c) $T\cos\theta = mg$
$T = \dfrac{10 \times 10}{\cos 30^\circ} = 115$ N

62.(b) $F = G\,\dfrac{x\,M(1-x)M}{r^2}$
For F to be maximum $\dfrac{dF}{dx} = 0$

or, $\frac{d}{dx}\left\{\frac{GM^2x(1-x)}{r^2}\right\} = 0$

or, $\frac{d}{dx}(x-x^2) = 0$

or, $1 = 2x$ or, $x = \frac{1}{2}$

63.(a) $K = \frac{P}{\frac{-\Delta v}{v}}$ if volume decreases then density increases

$\therefore \quad p = K\frac{\Delta\rho}{\rho}$

or, $\rho gh = K\frac{\Delta\rho}{\rho}$

or, $h = K\left(\frac{\Delta\rho}{\rho}\right)\frac{1}{\rho g}$

$= 10^8 \times \frac{0.5}{100} \times \frac{1}{10^3\times 10} = 50\text{m}$

64.(c) When a single drop of radius R is split into n drops of equal size then energy required is

$W = 4\pi R^2 T\,(n^{1/3} - 1)$

$\therefore \quad W \propto (n^{1/3} - 1)$

65.(c) $\mu = \frac{\sin\left(\frac{A+\delta_{min}}{2}\right)}{\sin\frac{A}{2}} = \frac{\sin\frac{2A}{2}}{\sin\frac{A}{2}}$

$= \frac{2\sin\frac{A}{2}\cos\frac{A}{2}}{\sin\frac{A}{2}}$

$\therefore \quad \mu = 2\cos\frac{A}{2}$

or, $\cos\frac{A}{2} = \frac{\mu}{2}$

or, $\frac{A}{2} = \cos^{-1}\left(\frac{\mu}{2}\right)$

or, $A = 2\cos^{-1}\left(\frac{\mu}{2}\right)$

66.(c) $\beta = \frac{D\lambda}{d}$ and, $\beta' = \frac{2D\lambda}{\frac{d}{2}} = 4\frac{D\lambda}{d} = 4\beta$

67.(c) $C_{rms}^{He} = C_{rms}^{H}$

or, $\sqrt{\frac{3RT_{He}}{M_{He}}} = \sqrt{\frac{3RT_H}{M_H}}$

or, $\frac{T_{He}}{4} = \frac{273}{2}$

or, $T_{He} = 546K = 273^0C$

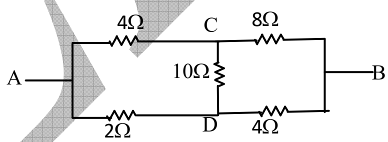
68.(c) $F = Kx$

$K = \frac{5000}{0.2} = 25000\text{N/m}$

$\frac{PE_{spring}}{PE_{capacitor}} = \frac{\frac{1}{2}Kx^2}{\frac{1}{2}CV^2}$

$= \frac{25000\times 0.2^2}{10\times 10^{-6}\times(10000)^2} = \frac{1000}{1000} = 1:1$

69.(b)



Here $\frac{R_{AC}}{R_{CB}} = \frac{R_{AD}}{R_{DB}}$, then C & D are at same potential due to which no current flows from C to D

$\therefore \frac{1}{R_{eq}} = \frac{1}{R_{AC}+R_{AB}} + \frac{1}{R_{AD}+R_{DB}} = \frac{1}{4+8} + \frac{1}{2+4}$

$\therefore \quad R_{eq} = \frac{12\times 6}{12+6} = 4\Omega$

70.(d) For dc

$R = \frac{v}{I} = \frac{160}{10} = 16\Omega$

<u>For Ac</u>

$I = \frac{V}{\sqrt{R^2 + X_L^2}}$

or, $16^2 + (2\pi fL)^2 = \left(\frac{200}{10}\right)^2$

or, $2\pi fL = \sqrt{400 - 256}$

$\therefore \quad L = \frac{12}{2\times3\times50} = 0.04H = 40mH$

71.(a) $KE = E - \phi = eV_s$

$\therefore \quad V_s = \frac{(4-2)eV}{e} = 2V$

72.(a) $1 - \left(\frac{N'}{N_o}\right) = \left(\frac{1}{2}\right)^{t/T_{\frac{1}{2}}}$

or, $1 - \frac{7}{8} = \left(\frac{1}{2}\right)^{t/5}$

or, $\left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^{t/5}$

or, t = 15 days

73.(b) $I = I_0 e^{-\mu x}$

or, $\frac{I_o}{4} = I_o e^{-\mu x}$

or, $\frac{1}{4} = \frac{1}{e^{\mu x}}$

or, $e^{\mu x} = 4$

or, $\mu x = \mathit{ln}4$

or, $\mu = \frac{\mathit{ln}\,4}{x} = \frac{\mathit{ln}4}{6.39} = 0.22$ mm

74.(a) The hydrocarbon formed is ethyne which on ozonolysis gives glyoxal.

75.(d) The gas formed is SO_2 which is reductant, oxidant and bleachant.

76.(c) $V_a \times N_a = V_b \times N_b$

$500 \times \frac{1}{10} \times 2 = V_b \times \frac{10 \times 10}{40}$

$\therefore \quad V_b = 40ml$

77.(d) For As_2S_3, $K_{sp} = 108S^5$

$S = \left(\frac{K_{sp}}{108}\right)^{1/5}$

$= \left(\frac{5 \times 10^{-36}}{108}\right)^{1/5}$

$= 3.41 \times 10^{-8}$ M

$[S^{--}] = 3 \times 3.41 \times 10^{-8}$

$= 1.02 \times 10^{-7}$ M

78.(c) NA H_2O molecules = 18g

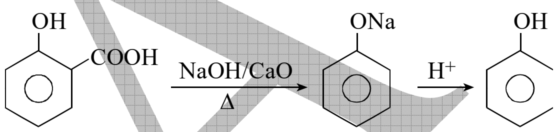
11.2 litres of N_2O gas at STP = 22g

0.5 moles of SO_2 = 32g

79.(c) The gas is acetylene which adds HCl to give 1-chloroethene.

80.(d) The gas formed is H_2S and given HgS precipitate in all medium as Hg^{++}-ions is group II metal ion.

81.(b)



82.(b) $\frac{b^2 + c^2 - a^2}{2bc} = \frac{1}{2} \Rightarrow b^2 + c^2 - a^2 = bc$

$\Rightarrow \quad b^2 - 2bc + c^2 = a^2 - bc$

$\Rightarrow \quad (b - c)^2 = a^2 - bc$

83.(d) Mean $(\overline{X}) = \frac{\Sigma fx}{N}$

$3 = \frac{5 + 8 + 18 + 4f}{15 + f}$

f = 14

84.(c) Polynomials are always continuous as **R**. If f(x), g(x) are both continuous, then so is $\frac{f(x)}{g(x)}$ except at those points where g(x) = 0. The given function is simply $\frac{(x-2)(x-3)}{(x-1)(x-3)}$.

85.(b) $\frac{dx}{dt} = 2t, \frac{dy}{dt} = 3t^2$. So, $\frac{dy}{dx} = \frac{3}{2}t$.

Now, $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right)\cdot\frac{dt}{dx}$

$= \frac{3}{2}\cdot\frac{1}{2t} = \frac{3}{4t}$

86.(a) $\frac{8!}{r!(8-r)!} = 2 \times \frac{8!}{(r-1)!\,(8-r+1)!}$

$\Rightarrow \frac{1}{r} = \frac{2}{8-r+1} \Rightarrow r = 3$

So, $^8C_3 = \frac{8 \times 7 \times 6}{3!} = 56$

87.(b) $a^x = b^y = c^z = k$

$\Rightarrow x\,l\text{na} = y\,l\text{nb} = z\,l\text{nc} = l\text{nk}$

Now, $y^2 = zx \Rightarrow \frac{(l\text{nk})^2}{(l\text{nb})^2} = \frac{l\text{nk}}{l\text{nc}} \cdot \frac{l\text{nk}}{l\text{na}}$

$\Rightarrow \frac{l\text{na}}{l\text{nb}} = \frac{l\text{nb}}{l\text{nc}}$

$\Rightarrow \log_b a = \log_c b$

88.(d) Total number ways = 2^n

If a head comes odd times then the favourable cases = 2^{n-1}

$P(E) = \frac{2^{n-1}}{2^n} = \frac{1}{2}$

89.(d) In the parabola $y^2 = -8x$, $a = -2$. So, comparing $y = -2x - k$ with $y = mx - 2am - am^3$, $m = -2$. So, $k = 2am + am^3 = 2 \times (-2) \times (-2) \times (-2)\,(-2)^3 = 8 + 16 = 24$.

Alternately, slope of the tangent is $\frac{1}{2}$.

From the parabola $\frac{dy}{dx} = -\frac{4}{y}$. Now, $-\frac{4}{y} = \frac{1}{2}$ gives $y = -8$. So, $(-8, -8)$ is the point of contact, which must satisfy the normal.

90.(d) It's an ellipse so,

$e = \sqrt{1 - \frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

91.(a) If $\sqrt{x} = t$, then $\frac{1}{2\sqrt{x}}\,dx = dt$.

So, the integral becomes, $2\int e^t.t\,dt = 2[te^t - \int e^t dt] = 2e^t[t-1] + c$

$= 2e^{\sqrt{x}}(\sqrt{x} - 1) + c$

92.(c) $\frac{dy}{dx} = 3x^2 - 6x = 0$ at $x = 0, 2$

When $x = 0$, $y = 1$ and when $x = 2$, $y = -3$

93.(a) $|\vec{u} \times \vec{v}| = |(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})| = 2\,|\vec{a} \times \vec{b}|$

Since,

$|\vec{a} \times \vec{b}|^2 + (\vec{a}.\vec{b})^2 = (ab\sin\theta)^2 + (ab\cos\theta)^2$
$= a^2b^2$

$|\vec{a} \times \vec{b}|$

$= \sqrt{a^2b^2 - (\vec{a}.\vec{b})^2} = \sqrt{4 \times 4 - \left(2 \times 2 \times \frac{1}{2}\right)^2}$

$= \sqrt{12} = 2\sqrt{3}$

94.(d) If $x = \tan\theta$, i.e. $\theta = \tan^{-1}x$, $y = \tan^{-1}\left(\frac{1 + \sec\theta}{\tan\theta}\right)$

$= \tan^{-1}\left(\frac{1 + \cos\theta}{\sin\theta}\right) = \tan^{-1}\left(\cot\frac{\theta}{2}\right) = \frac{\pi}{2} - \frac{\theta}{2}$

$= \frac{\pi}{2} - \frac{1}{2}\tan^{-1}x$. So, $\frac{dy}{dx} = -\frac{1}{2(1+x^2)}$

95.(c) $\int_0^{\pi/4} (\sec^2x - 1)\sec x \tan x\,dx = \int_1^{\sqrt{2}} (y^2 - 1)\,dy$

$= \left[\frac{y^3}{3} - y\right]_1^{\sqrt{2}} = \left[\frac{(\sqrt{2})^3}{3} - \sqrt{2}\right] - \left[\frac{1}{3} - 1\right]$

$= \frac{2\sqrt{2}}{3} - \sqrt{2} - \frac{1}{3} + 1 = \frac{2 - \sqrt{2}}{3}$

96.(d) The equation is equivalent to the system of linear equations $x - 2y + 3 = 0 = -2x + 4y + 5$ which is inconsistent.

97.(d) 98.(a) 99.(c) 100.(b)

...The End...