

Section - I

- 1.(a) 2.(b) 3.(c) 4.(d) 5.(d) 6.(d)
 7.(a) 8.(a) 9.(b) 10.(b) 11.(d) 12.(a)
 13.(a) Given equation is

$$x^2 + (k+2)x + 2k = 0$$
 For equal roots, $(k+2)^2 - 4 \cdot 1 \cdot 2k = 0$
 or, $k^2 + 4k + 4 - 8k = 0$
 or, $k^2 - 4k + 4 = 0$
 or, $(k-2)^2 = 0$
 i.e. $k = 2$
 14.(b) $t_n = 2n - 1$, $\Sigma t_n = (2n - 1) = n^2$
 15.(d) $A^2 - A + I = 0$
 $I = A - A^2$
 $A^{-1} = A^{-1}A - A^{-1}AA = I - A$
 16.(c) $A \cap B = C \Rightarrow C \subseteq A$
 $B \cap C = A \Rightarrow A \subseteq C$
 So, $A = C$
 17.(d) $\vec{i} \cdot (\vec{j} \times \vec{k}) + \vec{j} \cdot (\vec{k} \times \vec{i}) + \vec{k} \cdot (\vec{i} \times \vec{j})$
 $= \vec{i} \cdot \vec{i} + \vec{j} \cdot \vec{j} + \vec{k} \cdot \vec{k}$
 $= 1 + 1 + 1 = 3$
 18.(c) Product of slopes $= -\frac{1}{\sqrt{3}} \times -\frac{\sqrt{3}}{1} = -1$
 \therefore Angle between the lines $= 90^\circ$
 19.(d) Line $y = mx + c$ touches the circle $x^2 + y^2 = a^2$ if
 $a = \pm \frac{m \cdot 0 - 0 + c}{\sqrt{m^2 + (-1)^2}}$ i.e. $c = \pm a \sqrt{1 + m^2}$
 20.(b) By definition, eccentricity of circle $= 0$
 21.(a) Eqⁿ of plane cutting equal intercepts is $x + y + z = a$
 But it passes through $(2, 3, 4)$ So $a = 9$
 22.(d) $\tan^{-1} \tan \frac{2\pi}{3} = \tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3}$
 23.(c) A complex number $r(\cos\theta + i\sin\theta)$ is purely imaginary if $\cos\theta = 0$ i.e. $\theta = \frac{\pi}{2}$
 24.(c) Since $0 \leq b_{xy} \times b_{yz} \leq 1$
 So range is $[0, 1]$
 25.(c) $P(F \text{ or } S) = P(F) + P(S) - P(F \cap S) = \frac{2}{7} + \frac{2}{7} - \frac{1}{7} = \frac{3}{7}$
 26.(d) By definition logarithm is defined for positive values only so the region is $-1 < x \leq 1$
 $\lim_{x \rightarrow 0^+} \frac{|x|}{x} = \lim_{x \rightarrow 0^+} \frac{x}{x} = 1$
 27.(a) $\lim_{x \rightarrow 0^+} \frac{|x|}{x} = \lim_{x \rightarrow 0^+} \frac{x}{x} = 1$
 28.(d) $\log_{\sqrt{x}} x = 2 \log_x x = 2$ and its derivative is 0
 29.(c) Put $y = x + \log_e \sec x$ i.e. $dy = (1 + \tan x) dx$
 So $\int \frac{(1 + \tan x)}{x + \log_e \sec x} dx = \log_e(x + \log_e \sec x) + c$
 30.(b) For given sum, product is maximum when the numbers are equal. So $x = y = 5$.
 $xy = 5 \times 5 = 25$
 31.(a)
 32.(c) Required area $= \int_1^4 y dx = \int_1^4 \sqrt{x} dx$
 $= \left[\frac{x^{3/2}}{3/2} \right]_1^4$
 $= \frac{2}{3} [4^{3/2} - 1^{3/2}]$
 $= \frac{2}{3} \cdot 7 = \frac{14}{3}$ sq. units

- 33.(c) $R = \sqrt{A^2 + 2AB\cos\theta + B^2}$
 or, $A^2 + B^2 = A^2 + 2AB\cos\theta + B^2$
 or, $\cos\theta = 0 = \cos 90^\circ$
 or, $\theta = 90^\circ = \frac{\pi}{2}$
 34.(d) KE = workdone against friction
 or, $\frac{1}{2}mv^2 = f_f \times S$
 or, $\frac{1}{2}mv^2 = \mu mgS$
 or, $S = \frac{v^2}{2\mu g}$
 35.(d) $2T\cos\theta = \rho g h r$
 or, $h_1 r_1 = h_2 r_2$
 or, $h_1 \sqrt{\frac{A_1}{\pi}} = h_2 \sqrt{\frac{A_2}{\pi}}$
 or, $4 \times \sqrt{A} = h_2 \sqrt{\frac{A}{4}}$
 or, $h_2 = 8 \text{ cm}$
 36.(a) $I_0 = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\phi$
 $= I + 4I + 2\sqrt{I \times 4I} \cos 90^\circ$
 $= 5I$
 37.(b)
 38.(a) $du = nC_v dT = n \frac{R}{\gamma - 1} (75 - 25)$
 $= \frac{2 \times 50 \times R}{\gamma - 1}$
 $= \frac{100R}{\gamma - 1}$
 39.(b) $\sigma_1 = \sigma_2$
 or, $\frac{Q_1}{4\pi r_1^2} = \frac{Q_2}{4\pi r_2^2}$
 or, $\frac{Q_1}{4\pi \epsilon_0 r_1^2} = \frac{Q_2}{4\pi \epsilon_0 r_2^2}$
 or, $\frac{V_1}{r_1} = \frac{V_2}{r_2}$
 or, $\frac{V_1}{V_2} = \frac{r_1}{r_2}$
 40.(b) $\frac{R'}{R} = \left(\frac{1 + \frac{l}{10}}{l} \right)^2 = (1.1)^2 = 1.21$
 $\therefore R' = 1.21 \times 10 = 12.1 \Omega$
 41.(d) $E = \frac{1}{2}mv^2$
 $v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2 \times 2 \times 1.6 \times 10^{-13}}{1.67 \times 10^{-27}}}$
 $= 1.96 \times 10^7 \text{ m/s}$
 $\therefore F = Bev = 2.5 \times 1.6 \times 10 \times 1.96 \times 10^7$
 $= 7.8 \times 10^{-12} \text{ N}$
 42.(b) $\frac{H_1}{H_2} = \frac{\frac{V^2}{R_1}}{\frac{V^2}{R_2}} = \frac{R_2}{R_1}$
 43.(b) $i = 2r$
 $\mu = \frac{\sin i}{\sin r} = \frac{\sin 2r}{\sin r} = \frac{2\sin r \cos r}{\sin r}$
 or, $\cos r = \frac{\mu}{2}$

- or, $r = \cos^{-1}\left(\frac{\mu}{2}\right)$
 $\therefore i = 2r = 2\cos^{-1}\left(\frac{\mu}{2}\right)$
- 44.(b)** $C = \sin^{-1}\left(\frac{3}{5}\right)$
 or, $\frac{3}{5} = \sin C$
 or, $\mu = \frac{1}{\sin C} = \frac{5}{3}$
 $\therefore \tan i_p = \mu$
 or, $i_p = \tan^{-1}\left(\frac{5}{3}\right)$
- 45.(d)** $\alpha = 0.9$
46.(b) $\beta = \frac{\alpha}{1-\alpha} = \frac{\alpha}{1-0.9} = 9$
 $\beta = \frac{\Delta I_c}{\Delta I_b}$ or, $\Delta I_c = 9 \times 2 = 18 \mu A$
- 47.(a)** Let x be the required weight
 $0.05 = x/500 \times 1000/(174)$
 $= x \approx 4.355 \text{ gm}$
- 48.(d)** Since, H_2O_2 and H_2O are two compounds of 2 elements
- 49.(b)** $N/N_0 = (1/2)^t = \left(\frac{1}{2}\right)^{32/T_{1/2}}$
 $= T_{1/2} = 32/2 = 16 \text{ minutes}$
- 50.(c)** N_3H is hydrazoic acid that removes H^+ to give conjugate base N_3^-
- 51.(a)** Heliox, a mixture of helium and oxygen used for the treatment of asthma patient due to its density three times less than that of air.
- 52.(b)** In N_2O_5 , oxygen atom is bonded to two N- atoms.
- 53.(d)** ZnO is Chinese white
- 54.(a)** Phorone is obtained by condensation polymerization of three molecules of acetone in presence of HCl (g). PAN is polyacrylonitrile which is obtained by addition polymerization of acrylonitrile (vinyl cyanide)
- 55.(d)** Mesitylene is obtained by addition polymerization of propyne and condensation polymerization of acetone in presence of conc. H_2SO_4 .
- 56.(b)** Inductive effect : due to permanent displacement of σ - electrons
 Electromeric effect : complete transfer of electrons of multiple bond to one of the bonded atoms (usually more electronegative)
- 57.(b)** Cl^- is stronger leaving group than others.
- 58.(b)** $CH_2Cl - C(CH_3)_2 - CH_3$

Section - II

- 61.(c)** y is undefined when $x = 0$
 $\frac{x}{|x|} = \pm 1$
 So Range = $\{-1, 1\}$
- 62.(b)** We have $(1+x)^n = C_0 + C_1x + C_2x^2 + \dots + C_nx^n$

On differentiation, $n(1+x)^{n-1} = C_1 + 2C_2x + \dots + nC_nx^{n-1}$

Putting $x = 1$, $n2^{n-1} = C_1 + 2C_2 + \dots + nC_n$

63.(c)

64.(c)

$$\begin{aligned} & (1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8) \\ &= (1+\omega)(1+\omega^2)(1+\omega^3\omega)(1+(\omega^3)^2\omega^2) \\ &= (1+\omega)(1+\omega^2)(1+\omega)(1+\omega^2) \\ &= (1+\omega)^2(1+\omega^2)^2 \\ &= (-\omega^2)^2(-\omega)^2 \quad (\because 1+\omega+\omega^2=0) \\ &= \omega^4\omega^2 = (\omega^3)^2 = 1 \end{aligned}$$

65.(c)

$$|\vec{a}| = |\vec{b}| = |\vec{c}| = 1, \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$$

Now $|\vec{a} + \vec{b} + \vec{c}|$

$$\begin{aligned} &= \sqrt{(\vec{a} + \vec{b} + \vec{c})^2} \\ &= \sqrt{a^2 + b^2 + c^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})} \\ &= \sqrt{1+1+1+0} \\ &= \sqrt{3} \end{aligned}$$

66.(b)

$$\text{Eq}^n: ax^2 + 2hxy + by^2 = 0 \dots (i)$$

Let $y = mx$ be one of the lines represented by (i) which bisects the angle between the coordinate axes. Then $m = \pm 1$.

So putting (i) $ax^2 + 2hxy + b(\pm x)^2 = 0$

$$a \pm 2h + b = 0$$

or, $a + b = \pm 2h$

$$\therefore (a+b)^2 = 4h^2$$

67.(a)

Centre $(h, k) = (1, -3)$

$$r = \sqrt{(-2-1)^2 + (1+3)^2} = 5$$

Equation of circle is

$$\begin{aligned} & (x-h)^2 + (y-k)^2 = r^2 \\ \therefore & (x-1)^2 + (y+3)^2 = 25 \end{aligned}$$

68.(c)

69.(c) Given, $\frac{2b^2}{a} = 3$

or, $b^2 = \frac{3a}{2}$

And $e = \sqrt{1 - \frac{b^2}{a^2}}$

or, $\frac{1}{\sqrt{2}} = \sqrt{1 - \frac{3a}{2a^2}}$

or, $\frac{1}{2} = 1 - \frac{3}{2a}$

$\therefore a = 3$

$$b^2 = 3 \cdot \frac{3}{2} = \frac{9}{2}$$

The equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

or, $\frac{x^2}{9} + \frac{y^2}{\frac{9}{2}} = 1$

$\therefore x^2 + 2y^2 = 9$

70.(c)

At yz-plane $x = 0$

Let ratio be $k : 1$

Then $x = \frac{kx_2 + x_1}{k+1}$

- or, $0 = \frac{k(-1) + 4}{k + 1}$
 or, $k - 4 = 0$
 $k = 4$
 \therefore Ratio = 4 : 1
 Shortcut: By yz - plane ratio
 $= \frac{-x_1}{x_2} = \frac{-4}{-1} = 4:1$
- 71.(b)** We have $C = 45^\circ$, So $A + B = 135^\circ$
 Then $\cot(A + B) = \cot 135^\circ$
 $\frac{\cot A \cot B - 1}{\cot B + \cot A} = -1$
 or, $\cot A \cot B - 1 = -\cot A - \cot B$
 or, $\cot A \cot B - 1 + \cot A + \cot B = 0$
 i.e. $\cot A \cot B + \cot A + \cot B = 1$
 So $(1 + \cot A)(1 + \cot B) = 1 + \cot A + \cot B + \cot A \cot B = 1 + 1 = 2$
- 72.(b)** $f(x) = x^3 + \alpha x^2 + \beta x + 1$
 So $f'(x) = 3x^2 + 2\alpha x + \beta$
 Now $f'(0) = 0 \Rightarrow 3.0 + 2\alpha.0 + \beta = 0 \Rightarrow \beta = 0$
 $f'(1) = 0 \Rightarrow 3.1^2 + 2\alpha.1 = 0 \Rightarrow \alpha = -\frac{3}{2}$
- 73.(a)** $y = \sec^{-1} \frac{\sqrt{x} + 1}{\sqrt{x} - 1} + \sin^{-1} \frac{\sqrt{x} - 1}{\sqrt{x} + 1}$
 $= \cos^{-1} \frac{\sqrt{x} - 1}{\sqrt{x} + 1} + \sin^{-1} \frac{\sqrt{x} - 1}{\sqrt{x} + 1} = \frac{\pi}{2}$
 So $\frac{dy}{dx} = 0$
- 74.(c)** Given $\int_a^b x^3 dx = 0$
 $\left[\frac{x^4}{4} \right]_a^b = 0 \Rightarrow b^4 - a^4 = 0$
 $\Rightarrow b^4 = a^4$
 $\Rightarrow b = \pm a$
 and $\int_a^b x^2 dx = \frac{2}{3}$
 $\left[\frac{x^3}{3} \right]_a^b = \frac{2}{3}$
 $\frac{b^3 - a^3}{3} = \frac{2}{3}$
 $b^3 - a^3 = 2 \dots (ii)$
 When $b = a$, $b^3 - b^3 = 2 \Rightarrow 0 = 2$ (not possible)
 So, $b = -a$, $-a^3 - a^3 = 2 \Rightarrow a^3 = -1$
 $\Rightarrow a = -1$
 and $b = 1$
- 75.(d)** Solving $y = x$ with $y = x^2$, we get
 $x = 0, 1$
 Also the curves are symmetrical about y-axis
 So, required area
 $= 2 \int_0^1 (x - x^2) dx$
 $= 2 \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_0^1$
 $= 2 \left(\frac{1}{2} - \frac{1}{3} \right)$
 $= 2 \times \frac{1}{6} = \frac{1}{3}$ sq. units

- 76.(c)** $h_3 = u - \frac{a}{2} (2 \times 3 - 1)$
 $= 10 - \frac{2}{2} (6 - 1)$
 $= 5$ m
- 77.(c)** Workdone (W) = change in PE of hanging part
 $= \left(\frac{m}{l} \times x \right) g \times \frac{x}{2}$ (x = length of hanging part)
 $= \frac{4}{2} \times 0.6 \times 10 \times \frac{0.6}{2}$
 $= 3.6$ J
- 78.(b)** $PA = 2\pi r.T$
 or, $\rho gh \times \pi r^2 = 2\pi r.T$
 or, $r = \frac{2T}{\rho gh}$
 $\therefore d = 2r = \frac{4T}{\rho gh} = \frac{4 \times 0.07}{10^3 \times 10 \times 0.4}$
 $= 7 \times 10^{-5}$ m
 $= 0.07$ mm
- 79.(d)** For 1st
 $n_1 = \frac{1}{2l} \sqrt{\frac{T}{\pi(2r)^2 \rho}} = \frac{1}{4lr} \sqrt{\frac{T}{\pi \rho}}$
 2nd $n_2 = \frac{1}{2 \times 2l} \sqrt{\frac{T}{\pi r^2 \rho}} = \frac{1}{4lr} \sqrt{\frac{T}{\pi \rho}}$
 $\therefore n_1 : n_2 = 1 : 1$
- 80.(b)** Heat lost by block = Heat gained by ice
 or, $2 \times S \times 500 = mL_f$
 or, $m = \frac{2 \times 400 \times 500}{80 \times 4200} = 1.2$ kg
- 81.(c)** 1st case
 $\eta_1 = \left(1 - \frac{T_2}{T_1} \right) \times 100\%$
 $\frac{40}{100} = \left(1 - \frac{300}{T_1} \right)$
 or, $\frac{300}{T_1} = 1 - \frac{2}{5} = \frac{3}{5}$
 or, $T_1 = 500$ K
 2nd case
 $\eta_2 = \left(1 - \frac{T_2}{T_1} \right) \times 100\%$
 or, $\frac{60}{100} = 1 - \frac{300}{T_1}$
 or, $\frac{300}{T_1} = 1 - \frac{2}{5} = \frac{3}{5}$
 or, $T_1' = 750$ K
 $\Delta T = T_1' - T_1 = 750 - 500$
 $= 250$ K
- 82.(a)** Given,
 $F_1 = 9$ N $F_2 = ?$
 $r_1 = d$ $r_2 = 3d$
 $\therefore F \propto \frac{1}{r^2}$
 $\therefore \frac{F_2}{F_1} = \left(\frac{r_1}{r_2} \right)^2$
 or, $F_2 = \left(\frac{d}{3d} \right)^2 \times F_1$
 $F_2 = \frac{1}{9} \times 9$ N
 $\therefore F_2 = 1$ N

- 83.(b)** Across AC
 $R' = \frac{18 \times 6}{18 + 6}$
 $= \frac{18 \times 6}{244} = 4.5 \Omega$
 $R_T = R' + r = (4.5 + 1.5) = 6\Omega$
 $I = \frac{E}{R_T} = \frac{18}{6} = 3A$
 Now $I' \times 18 = (3 - I') \times 6$
 or, $3I' = 3 - I'$
 or, $I' = \frac{3}{4} = 0.75A$
- 84.(c)** KE of electron = 10 eV
 $\therefore \frac{1}{2}mv^2 = 10 \times 1.6 \times 10^{-19}$
 $V = 1.875 \times 10^6 \text{ m/sec}$
 and $\frac{mV^2}{r} = eVB$
 $r = \frac{mV}{eB} = 11 \times 10^{-2} \text{ m} = 11 \text{ cm}$
- 85.(a)** $u + v = x$
 $v = x - u$
 And $m = \frac{v}{u}$
 or, $v = mu$
 So $mu = x - u$
 or, $u = \frac{x}{m + 1}$
 & $v = x - \frac{x}{m + 1} = \frac{(m + 1)x - x}{m + 1}$
 $= \frac{mx + x - x}{m + 1} = \frac{mx}{m + 1}$
 $\therefore f = \frac{uv}{u + v} = \frac{\left(\frac{x}{m + 1}\right) \left(\frac{mx}{m + 1}\right)}{\frac{x}{m + 1} + \frac{mx}{m + 1}}$
 $= \frac{\frac{mx^2}{(m + 1)^2}}{\frac{(m + 1)x}{(m + 1)^2}}$
 $= \frac{mx^2}{(m + 1)^2 \times x}$
 $= \frac{mx}{(m + 1)^2}$
- 86.(c)** $\frac{x}{f} = \frac{\lambda}{d}$
 or, $x = \frac{f\lambda}{d}$
- $\therefore 2x = \frac{2f\lambda}{d} = \frac{2 \times 0.5 \times 6000 \times 10^{-10}}{0.2 \times 10^{-3}}$
 $= 3 \times 10^{-3} \text{ m}$
 $= 3 \text{ mm}$
- 87.(b)** $\phi = 1.24 \text{ eV}$
 $KE = \frac{hc}{\lambda} - \phi$
 $= \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{4.36 \times 10^{-7}} - 1.24 \times 1.6 \times 10^{-19}$
 $= 2.57 \times 10^{-19} \text{ J}$
 $\therefore KE = eV_s$
 or, $V_s = \frac{KE}{e} = \frac{2.57 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.6 \text{ V}$
- 88.(a)** For S_1
 $A_1 = -\lambda_1 2N_0$
 $5 \times 10^{-6} = \frac{0.693}{T_{1/2}} 2N_0 \dots (i)$
 For S_2
 $A_2 = -\lambda_2 N_0$
 or, $10 \times 10^{-6} = -\frac{0.693}{T_{1/2}} \times N_0 \dots (ii)$
 Dividing (i) by (ii)
 $\frac{5 \times 10^{-6}}{10 \times 10^{-6}} = \frac{2}{T_{1/2}} \times T_{1/2}'$
 or, $\frac{T_{1/2}}{T_{1/2}'} = 20 : 5$
- 89.(d)** O.N. of S in $SO_3^{2-} = +4$
90.(b) O.N. of S in $S_2O_6^{2-} = +5$
 O.N. of S in $S_2O_4^{2-} = +3$
91.(c) In $CaCl_2$, $[Cl^-] = 2 \times 0.04 = 0.08 \text{ mol/L}$
 $[Cl^-]$ from $AgCl$ is very low so it is neglected
 $K_{sp} = [Ag^+][Cl^-]$
 $4 \times 10^{-10} = [Ag^+] \times 0.08$
 $\therefore [Ag^+] = 4 \times 10^{-10} / 0.08 = 5 \times 10^{-9} \text{ M}$
- 92.(b)** Relative atomic mass = $\frac{5 \times 54 + 90 \times 56 + 5 \times 57}{100}$
- 93.(c)** $MnO_4^- + 8H^+ + 5e^- \rightarrow Mn^{2+} + 4H_2O \times 2$
 $C_2O_4^{2-} \rightarrow 2CO_2 + 2e^- \times 5$
 $2MnO_4^- + 5C_2O_4^{2-} + 16H^+ \rightarrow 2Mn^{2+} + 10CO_2 + 8H_2O$
- 94.(b)** In $[Ni(CN)_4]^{2-}$, Ni has +2 oxidation number. CO and CN^- are strong ligands that result inward pairing of electrons and hence $[Ni(CN)_4]^{2-}$ has dsp^2 hybridization.
- 95.(d)** Secondary prefixes are arranged in alphabetical order
- 96.(c)** 4-primary amines, 3-secondary amines and 1-tertiary amine (draw structures or use NS trick)
- 97.(c)** **98.(c)** **99.(a)** **100.(b)**

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